An on-line supplement to 'A class of Lévy process models with almost exact calibration to both barrier and vanilla FX options'

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Abstract

This document is an on-line supplement to a recent paper (Carr and Crosby (2008)) entitled "A class of Lévy process models with almost exact calibration to both barrier and vanilla FX options".

OLS 1. Introduction

This document is an on-line supplement to Carr and Crosby (2008). Section 2 describes some of the details of the numerical implementation of the algorithms in Carr and Crosby (2008). Section 3 describes some examples and tests, partly in order to illustrate some of the features of the modelling framework but mainly to benchmark the accuracy of the algorithms. Section 4 provides the market data as of 6th July 2007 for cable (USD/STG) and more details on the calibration to that market data performed in Carr and Crosby (2008). Section 5 provides the market data as of 31st May 2007 for cable (USD/STG) and more details on the corresponding calibration performed in Carr and Crosby (2008). The references section includes both references used in Carr and Crosby (2008) and one additional reference (Wang et al. (2008)) used only in this on-line supplement. All equation numbers in this document refer to equations in Carr and Crosby (2008). We have used the prefix OLS (for on-line supplement) for all section numbers and table numbers in this document in order to distinguish them from section numbers and table numbers in Carr and Crosby (2008).

OLS 2. Numerical implementation of the algorithms

Our algorithms make frequent use of Laplace Transform inversion. We used the Gaver-Stehfest algorithm (see, for example, Kou and Wang (2003) and Sepp (2004) and the references therein). The advantages of this algorithm are that it is simple to implement, extremely fast and that it performs the Laplace inversion on the real-line. The disadvantage is that it, ideally, requires the use of high-precision arithmetic. Our experience is that one can often get away with standard double precision (15-16 significant figures of accuracy) but, equally, one does sometimes get numerical instabilities, when using standard double precision, especially for more extreme parameter values. Unless otherwise stated, our calculations used "quad double" precision (that is to say, using 60-64 significant figures of accuracy instead of the usual double precision). The code to do this "quad double" precision arithmetic was written by Alan Ambrose. His code was in turn based on work by David Bailey (Bailey (1990)) and Siddhartha Chatterjee (Chatterjee (1998)). The program was written in C# which is a compiled language broadly similar to C++ although typically somewhat slower. The Gaver-Stehfest algorithm controls accuracy via the number of terms, which we call nTerms for brevity, used in the series. A typical value of nTerms is 16 although, in the tests in section OLS 3, we variously used values between 12 and 20.

The numerical integrations required to perform the integrals in appendix 1 of Carr and Crosby (2008) used Bode's rule. Accuracy in the numerical integration is controlled by the number of points used, which we call numPoints for brevity. A typical value of numPoints was 2^{18} which is 262,144 although, in the tests in section OLS 3, we variously used values between 2^{17} and 2^{22}. This might seem like a lot of points but we need the integrals to be performed very accurately, otherwise we may introduce numerical instabilities in the Laplace inversion. The upper limits in the integrals are infinity. Therefore, clearly, we have to choose where to truncate these integrals. Observing the forms of the integrals in appendix 1 of Carr and Crosby (2008), we see that the integrands are all very similar. We wished to exploit this fact in order to speed up the calculations. For the purposes of calibration, we would typically like to price vanilla options with several (for example, five) different strikes simultaneously and the only term which depends on the strike is the term $\left(K/S(t_0)\right)^{\bar{j}z+1}$ (recall $\bar{j} = \sqrt{-1}$). In addition, the only term which depends on α , the Laplace Transform parameter, is $\Omega(z)$. With the choice $z = u + \bar{j}/2$, where u is real, the term $\left(z^2 - \bar{j}z\right) = u^2 + (1/4)$ and this term is common to all the integrals. The integrals to compute $L\left[I_{i,1\le i\le M/2}\right]$ and $L\left[I_{i,1+M/2\le i\le M}\right]$ involve additional terms $\left(\rho_i b_i + \bar{j}z\right)$ and $\left(-\rho_i b_i - \bar{j}z\right)$ respectively. It is straightforward to see that all $M+2$ integrands converge, in modulus, to zero for large u, at least as fast as $\Omega(u+\bar{j}/2)/\left(u^2 + (1/4)\right)$. We chose the value of u at which we truncated the upper limit of all these integrals by searching for the smallest value of u , $u_{\max}$, say, which made:

$$\max\left(\operatorname{Re}\left(\Omega(u+\bar{j}/2)\frac{\left(K/S(t_0)\right)^{\bar{j}u}}{\left(u^2+(1/4)\right)}\right), \operatorname{Re}\left(\Omega(u+\bar{j}/2)\frac{\left(K/S(t_0)\right)^{\bar{j}u}}{\left(u^2+(1/4)\right)\left(\rho_i b_i + \bar{j}(u+\bar{j}/2)\right)}\right),\right.$$
$$\left.\operatorname{Re}\left(\Omega(u+\bar{j}/2)\frac{\left(K/S(t_0)\right)^{\bar{j}u}}{\left(u^2+(1/4)\right)\left(-\rho_i b_i - \bar{j}(u+\bar{j}/2)\right)}\right), \operatorname{Re}\left(\Omega(u+\bar{j}/2)\frac{1}{\left(u^2+(1/4)\right)}\right)\right),$$

less than some small tolerance, which we call convergenceTol for brevity (a typical value of convergenceTol was 1.0E-12), for all values α used in the Gaver-Stehfest algorithm and over all values of the subscript i, $i = 1,\ldots,M$ and over all values of K, where K refers to the different strikes of the vanilla options. This means that we were using the most conservative possible value of $u_{\max}$ (however, because the integrands, for large u, are only weakly dependent upon α and K, it was not so conservative so as to be inefficient). We then used this value of $u_{\max}$ for all the integrals we needed to perform. This meant that we could use common values of u when numerically integrating using Bode's rule. The total number of integrals we need to perform equals nTerms multiplied by $M+2$ multiplied by the number of different strikes (typically, five). Hence, this number might, for example, be 16 times 6 times 5. The number of integrand evaluations required will be this number multiplied by numPoints. Hence, this total will be very large (typically around 50 to 100 million). However, because we used common values of u and many of the various terms in the integrands are common, we saved these various terms in cache after the first time we had evaluated them and then retrieved them later. This resulted in a very considerable reduction in total computation time (typically, close to two orders of magnitude) compared to computing (each term in) each integrand separately.

Since it was not possible to compute the values of the integrals $L[I_0]$, $L\left[I_{i,1\le i\le M/2}\right]$, $L\left[I_{i,1+M/2\le i\le M}\right]$ and $L\left[I_{M+1}\right]$ to better accuracy than approximately 1.0E-12, with typical values of numPoints and convergenceTolerance, we used standard double precision when evaluating these integrals numerically by Bode's rule. Computations with "quad double" precision are typically two orders of magnitude slower than those with standard double precision and given the degree of accuracy in the numerical integrations, we decided it was not worth the extra computation time. This decision was also supported by the impressive accuracy of the results of tests 3 and 4 in section OLS 3 and by tables OLS 12 and OLS 13 in section OLS 4. We should stress, however, that we did use "quad double" precision in computing everything else, including the computation of the $(M+2)$ "probability like" terms in equation (4.6). Furthermore, we used "quad double" precision throughout all the computations when we calculated barrier option prices using equations (A.21) to (A.24) and when we calculated the terms $L[I_0]$, $L[I_{i,1\le i\le M/2}]$, $L[I_{i,1+M/2\le i\le M}]$ and $L[I_{M+1}]$ analytically using the results of appendix 3 (see remark 2.7 of Carr and Crosby (2008)) for the special case when the dynamics of the spot FX rate after the first exit time from the corridor are those of our CEE2 process.

All the results in sections OLS 3 and OLS 4 were computed on a laptop computer with clock speed 2GMHz and 2 GB of RAM.

OLS 3. Examples and tests of the accuracy of the algorithms

In this section, we describe a series of examples and tests that we did in order to test the integrity of our implementation, to benchmark the accuracy of our algorithms and to illustrate some of the features of the modelling framework. In addition, we hope that these tests may be useful to researchers or practitioners who wish to implement or extend our work. The examples and tests fall broadly into two categories – those that test the algorithm for pricing DNT options and double barrier knockout options (equations (A.21) to (A.24)) and those that test the algorithm for pricing double barrier knock-in options (equation (4.6)) and, hence, also vanilla options.

Before describing the tests, we should mention that some of our tests involved comparing the prices of DNT options and double barrier knockout options obtained via equations (A.21) to (A.24) with those obtained by Monte Carlo simulation. Our Monte Carlo simulation used the methodology described in appendix 4 of Carr and Crosby (2008). Specifically, we simulated the transition times of the Markov chain and then the jump times of the Poisson processes. In between these times, the log of the spot FX rate evolves as Brownian motion. We price DNT options or double barrier knockout options, by using results in Potzelberger and Wang (2001) concerning the probability of a Brownian bridge process hitting either of the barrier levels. This eliminates discretization error bias when pricing barrier options with continuously monitored barriers. Hence, we obtained unbiased estimates of the relevant barrier option prices which we can directly compare with the results obtained from equations (A.21) to (A.24).

We now proceed to describe some of the tests that we performed. Note that when we were coding our algorithms, we actually performed many more tests – the ones we describe here are illustrative rather than exhaustive. Our notation is defined in Carr and Crosby (2008).

Test 1:

We priced a DNT option, at time t, with the following parameters: $S(t)=220$, $r_d=0.046$, $r_f=0.051$, $L=180$, $U=260$, $T-t=0.9$. There were a total of four Poisson processes i.e. $M=4$ and the reciprocals of the mean jump sizes were: $b_1=50$, $b_2=25$, $b_3=25$, $b_4=50$. Furthermore, $a_1(1)=a_1(2)=0.9$, $a_2(1)=a_2(2)=0.45$, $a_3(1)=a_3(2)=0.51$, $a_4(1)=a_4(2)=0.84$, $\sigma(1)=\sigma(2)=0.23$. Note that, effectively, the Markov chain had only one state since the diffusion volatilities and the jump intensity rates were the same in each state. In terms of our general pricing formula (equations (A.21) to (A.24)), we priced the DNT option by setting $K=L=180$, $\varphi=1$ and $R_L=R_U=0$ and using a binary cash-or-nothing style payoff. We priced the DNT option and obtained a price of 0.1904052. We then priced two single-touch double barrier knockout options. The first option paid a rebate of 2.5 if the lower barrier was hit first prior to maturity and zero if the upper barrier was hit first prior to maturity. The second option paid a rebate of 2.5 if the upper barrier was hit first prior to maturity and zero if the lower barrier was hit first prior to maturity. The rebates were payable at maturity (not at the time the relevant barrier was touched or breached). If the spot FX rate did not touch or breach the lower barrier nor the upper barrier prior to maturity, then both single-touch double barrier knockout options expired with zero terminal payoff. We priced the first option by setting $K=L=180$, $\varphi=-1$, $R_L=2.5$, $R_U=0$ and we priced the second option by setting by setting $K=L=180$, $\varphi=-1$, $R_L=0$,

$R_U = 2.5$. We obtained prices of 0.9566707 and 0.9659295 respectively. If we form a portfolio of 2.5 DNT options and one of each of the single-touch double barrier knockout options, then it is clear that we have a portfolio that pays 2.5 units of domestic currency in all states of the world and hence must have the same price as the discounted value of 2.5. The former has a price of 2.3986132 and the latter has a price of $2.5\exp(-r_d(T-t)) = 2.3986132$ and hence the results are consistent. We also priced a double barrier knockout option which had zero terminal payoff if the spot FX rate did not touch or breach the lower barrier nor the upper barrier prior to maturity and paid a rebate of 2.5 if either barrier was touched or breached prior to maturity. The rebate was again payable at maturity. By setting $K = L = 180$, $\varphi = -1$, $R_L = 2.5$, $R_U = 2.5$ in our general pricing formula (equations (A.21) to (A.24)), we obtained a price of 1.9226002. This should, of course, equal the sum of the prices of the two single-touch double barrier knockout options obtained above. Since, $0.9566707 + 0.9659295 = 1.9226002$, the results are, again, consistent. For the prices reported above, the number of terms (nTerms) used in the Gaver-Stehfest algorithm was 16. Evaluating the prices of all four options considered in this test took approximately 1.2 seconds or an average of approximately 0.3 seconds per option. Using 14 terms, 18 terms or 20 terms, gave option prices consistent with those reported above with, occasionally, the prices changing in the last decimal place.

Test 2:

This test is essentially identical to an example in Kou and Wang (2004). We priced two options, at time t, using equations (A.21) to (A.24), within the Kou (2002) DEJD model (there are no changes in the dynamics of the spot FX rate at any corridor levels). The parameters were as follows: $S(t) = 100$, $r_d = 0.05$, $r_f = 0.0$, $T - t = 1$. For the Kou (2002) DEJD model, $M = 2$, and furthermore we have $b_1 = 50$, $b_2 = 25$, $a_1(1) = a_1(2) = 0.9$, $a_2(1) = a_2(2) = 2.1$, $\sigma(1) = \sigma(2) = 0.2$. We priced two double barrier knockout call options, both with a strike of 100. The first had barrier levels of $L = 1$, $U = 10000$ and the second had barrier levels of $L = 1$, $U = 120$. The number of terms (nTerms) used in the Gaver-Stehfest algorithm was 16. We obtained prices of 11.09365 and 1.04057 respectively. If we note the barrier levels in relation to the initial spot FX rate, we realise that we are, effectively, pricing a vanilla call option and an up-and-out call option respectively. If we take the difference, we obtain the price of an up-and-in call option to be $11.09365 - 1.04057 = 10.05307$ (allowing for rounding). In section 4.3 of Kou and Wang (2004), they price an up-and-in call option, with the same parameters, and obtain a price of 10.05307. Hence the results are consistent to seven significant figures and to five decimal places.

Test 3:

Suppose that the Markov chain effectively only has one state. We can price vanilla options when the dynamics of the spot FX rate follow our CEE2 process (equation (2.1)) in four different ways:

Case 1: We can price double barrier knockout options using equations (A.21) to (A.24). Then we can price double barrier knock-in options using the results of appendix 1, together with numerical integrations, where the parameters of the CEE2 process are exactly the same as those we used to price the double barrier knockout options. Then we add the two prices together. By "in-out" parity, this must give the price of a vanilla option where the dynamics of the spot FX rate follow our CEE2 process with the same parameters regardless of whether the corridor has been exited or not.

Case 2: We can do exactly as in case 1 but now we price the double barrier knock-in options using our analytical results for the CEE2 process (see remark A.7 of Carr and Crosby (2008)).

Case 3: We can price vanilla options under our CEE2 process, where the dynamics of the spot FX rate follow our CEE2 process with the same parameters regardless of whether the corridor has been exited or not, using the special case of equations (A.21) to (A.24) (see proposition A.5) applicable to vanilla options.

Case 4: We can price vanilla options under our CEE2 process, where the dynamics of the spot FX rate follow our CEE2 process with the same parameters regardless of whether the corridor has been exited or not, using Fourier inversion (see, for example, Lipton (2001), Lewis (2001) and Kangro et al. (2004)) and the form of the characteristic function for our CEE2 process.

Hence, we have four possible ways of pricing vanilla options. If our algorithms have been correctly implemented, then the results should agree (to within reasonable tolerances that depend on the parameters that control the accuracy of the algorithms such as nTerms and numPoints).

We priced options, at time t, with the following parameters: $S(t)=100$, $r_d=0.05$, $r_f=0.08$, $L=80$, $U=120$, $T-t=1.2$. There were a total of four Poisson processes i.e. $M=4$ and the reciprocals of the mean jump sizes were $b_1=67$, $b_2=38$, $b_3=36$, $b_4=81$. Furthermore the jump intensity rates were $a_1(1)=a_1(2)=1.03$, $a_2(1)=a_2(2)=2.1$, $a_3(1)=a_3(2)=1.9$, $a_4(1)=a_4(2)=1.21$ and the diffusion volatility was $\sigma(1)=\sigma(2)=0.09$. We priced put options with strikes of 95 and 100 and call options with strikes of 100, 105 and 110. For cases 1, 2 and 3, the number of terms (nTerms) used in the Gaver-Stehfest algorithm was set equal to 16.

Case 1: We priced double barrier knockout options using equations (A.21) to (A.24). We obtained prices of 1.846521, 3.792714, 1.651479, 0.673454, 0.191826 (for respectively, put (strike 95), put (strike 100), call (strike 100), call (strike 105), call (strike 110)). We then priced double barrier knock-in options, by numerical integration (as in appendix 1), using values of nTerms, numPoints and convergenceTol of 16, 21 and 2.5E-13 respectively. We obtained prices of 2.242524, 2.927891, 1.739075, 1.312807, 0.915265 respectively. By "in-out" parity, this implies vanilla prices of (allowing for rounding) 4.089044, 6.720605, 3.390554, 1.986261, 1.107091 (for respectively, put (strike 95), put (strike 100), call (strike 100), call (strike 105), call (strike 110)).

Case 2: We priced double barrier knock-in options, using our analytical results for the CEE2 process (see remark A.7 of Carr and Crosby (2008)). We obtained prices of 2.242524, 2.927891, 1.739075, 1.312807, 0.915265 respectively. These prices agree to six decimal places with those in case 1. The pricing of double barrier knockout options is as in case 1. Therefore, by "in-out" parity, this again implied vanilla prices of (allowing for rounding) 4.089044, 6.720605, 3.390554, 1.986261, 1.107091 (for respectively, put (strike 95), put (strike 100), call (strike 100), call (strike 105), call (strike 110)).

Case 3: We priced vanilla options using equations (A.21) to (A.24) in the special case for vanilla options. We obtained prices of 4.089044, 6.720604, 3.390552, 1.986263, 1.107090 (for respectively, put (strike 95), put (strike 100), call (strike 100), call (strike 105), call (strike 110)).

Case 4: We priced vanilla options using the Fourier inversion methodology of, for example, Lipton (2001), Lewis (2001) and Kangro et al. (2004) and the form of the characteristic function for our CEE2 process. We obtained prices of 4.089044, 6.720604, 3.390552, 1.986263, 1.107088 (for respectively, put (strike 95), put (strike 100), call (strike 100), call (strike 105), call (strike 110)).

Note that all the results agree to five decimal places and most agree to six decimal places. Hence, we can see that all four possible ways of pricing vanilla options give essentially the same answers.

Test 4:

In this test, we priced double barrier knock-in options using the results in appendix 1. We assumed that the dynamics of the spot FX rate before the first exit time from the corridor were those of our CEE2 process with a total of two Poisson processes and, effectively, only one state in the Markov chain. The parameters used were $b_1=64$, $b_2=64$, $a_1(1)=a_1(2)=0.000001$, $a_2(1)=a_2(2)=0.000001$, $\sigma(1)=\sigma(2)=0.19$. After the first exit time from the corridor, we assumed that the spot FX rate followed a CGMY process with parameters $C=16.97$, $G=7.08$, $M=29.97$, $Y=0.6442$ (and no Brownian component). We priced a double barrier knock-in call option, at time t, with the following additional data: $S(t)=90$, $r_d=0.06$, $r_f=0.0$, $L=89.998$, $U=90.002$, $T-t=0.25$, $K=98$. Given how close the corridor levels are to the initial spot FX rate, the spot FX rate will most probably exit from the corridor almost immediately.

Hence, the price of the double barrier knock-in option should be almost the same as that of a vanilla option under the CGMY process. Wang et al. (2008) price a vanilla option, under the CGMY process, with these same parameters using a PIDE methodology and obtain a price of approximately 16.212. We computed the vanilla option price using the Fourier inversion methodology of, for example, Lipton (2001), Lewis (2001) and Kangro et al. (2004), together with the form of the characteristic function for the CGMY process, and obtained a value of 16.211904 (correct to six decimal places). We computed the price of the double barrier knock-in call option using the results in appendix 1 for various values of the number of terms in the Gaver-Stehfest algorithm (labelled nTerms), the number of points (numPoints) used in the numerical integration (labelled log2(numPoints) - this is displayed as log, base 2, hence, for example, 17 means we used 2^{17} points) and the convergence tolerance (labelled convergenceTol) used to truncate the upper limits in the integrals. The results are displayed in table OLS 1. Again, we excellent agreement between the prices obtained by the different methodologies and we see good agreement between the prices obtained for different values of nTerms, numPoints and convergenceTol.

Table OLS 1

16.211904	Vanilla price		
Vanilla price	nTerms	log2(numPoints)	convergenceTol
16.211912	14	17	2.00E-12
16.211898	14	18	2.00E-12
16.211901	14	18	1.00E-12
16.212030	16	18	1.00E-12
16.212458	16	19	1.00E-12
16.211407	16	20	1.00E-12
16.211816	16	20	5.00E-13
16.211852	16	21	5.00E-13
16.210980	16	22	2.50E-13
16.213077	18	19	2.00E-12
16.219692	18	20	1.00E-12
16.211800	18	21	5.00E-13

Test 5:

All of our tests thus far have only considered the simpler case when there is effectively only one state in the Markov chain. This test will consider a richer specification when there are two distinct states in the Markov chain and, furthermore, there are a total of eight Poisson processes i.e. $M = 8$. The other parameters (which are just for illustration) were as follows: The reciprocals of the mean jump sizes were $b_1 = 80$, $b_2 = 60$, $b_3 = 40$, $b_4 = 20$, $b_5 = 20$, $b_6 = 40$, $b_7 = 60$, $b_8 = 80$. Furthermore the jump intensity rates were $a_1(1) = 0.9$, $a_2(1) = 0.45$, $a_3(1) = 0.12$, $a_4(1) = 0.06$, $a_5(1) = 0.14$, $a_6(1) = 0.17$, $a_7(1) = 0.51$, $a_8(1) = 0.84$, $a_1(2) = 1.1$, $a_2(2) = 0.65$, $a_3(2) = 0.22$, $a_4(2) = 0.09$, $a_5(2) = 0.21$, $a_6(2) = 0.24$, $a_7(2) = 0.71$, $a_8(2) = 1.04$ and the diffusion volatilities were $\sigma(1) = 0.05$, $\sigma(2) = 0.09$. We priced four double barrier knockout options, at time t, with four different terminal payoffs and the following additional data: $S(t) = 220$, $r_d = 0.046$, $r_f = 0.051$, $L = 195$, $U = 250$, $T - t = 0.9$, $K = 218$. All four options paid a rebate at maturity equal to 0.25 units of domestic currency if either barrier was hit prior to maturity i.e. $R_L = R_U = 0.25$. The four options had four different terminal payoffs if neither barrier was hit prior to maturity. These four payoffs were those of a vanilla call, a vanilla put, a call with a binary cash-or-nothing style (BCON) payoff and a put with a binary cash-or-nothing style (BCON) payoff. We priced each of the four options, when the initial state of the Markov chain was $\Psi(t) = 1$, when the initial state of the Markov chain was $\Psi(t) = 2$, when the Markov chain transition rates were $\varepsilon_{12} = 1.85$, $\varepsilon_{21} = 1.1$ and when the Markov chain transition rates were $\varepsilon_{12} = 10$, $\varepsilon_{21} = 10$. Hence, using equations (A.21) to (A.24), we generated sixteen different option prices. The results are displayed in table OLS 2, where we also show the prices, and below them the corresponding standard errors, obtained by Monte Carlo simulation with four million runs. Two comments are in order: Firstly, we see excellent agreement between the option prices obtained from equations (A.21) to (A.24) and those obtained by Monte Carlo simulation. Secondly, we see that the differences between the prices in the third and fourth columns of the tables are much smaller than the differences between the prices in the first and second columns. This is intuitive because in the third and fourth columns we have much higher Markov chain transition rates and hence it should make much less difference which was the initial state of the Markov chain. For all the prices in table OLS 2, the number of terms (nTerms) used in the Gaver-Stehfest algorithm was set equal to 12.

Table OLS 2

Prices obtained from equations (A.21) to (A.24)

Initial state	1	2	1	2
Markov chain trans. rates	1.85 / 1.1	1.85 / 1.1	10 / 10	10 / 10
Vanilla Call	4.39899	4.00910	4.40469	4.33799
Vanilla Put	2.83956	2.52770	2.81620	2.76183
BCON Call	0.46099	0.43320	0.45554	0.45108
BCON Put	0.38718	0.36507	0.38284	0.37919

Prices obtained from Monte Carlo simulation with four million runs

Initial state	1	2	1	2
Markov chain trans. rates	1.85 / 1.1	1.85 / 1.1	10 / 10	10 / 10
Vanilla Call	4.39849	4.00505	4.40590	4.33903
Vanilla Put	2.84134	2.52884	2.81717	2.76611
BCON Call	0.46093	0.43309	0.45547	0.45111
BCON Put	0.38724	0.36511	0.38312	0.37918

Standard errors for the Monte Carlo prices above

Initial state	1	2	1	2
Markov chain trans. rates	1.85 / 1.1	1.85 / 1.1	10 / 10	10 / 10
Vanilla Call	0.00340	0.00334	0.00344	0.00342
Vanilla Put	0.00245	0.00235	0.00245	0.00244
BCON Call	0.00022	0.00021	0.00022	0.00021
BCON Put	0.00021	0.00020	0.00021	0.00021

OLS 4. Calibration to market data as of 6th July 2007

In this section, we describe in more detail the calibration to market data for cable (USD/STG) as of 6th July 2007 and provide the market data used.

The market data as of 6th July 2007 used for the calibration was as follows: The (initial) spot FX rate (number of USD per STG) was $S(t_0) = 2.0060$. The corridor levels were $L = 1.9500$ and $U = 2.0500$. The interest-rates (continuously compounded) were as in table OLS 3.

Table OLS 3

Tenor	Time in years	Interest-rates are continuously compounded USD interest-rates	STG interest-rates
1 w	0.019178082192	0.053475001288	0.058852668688
1 m	0.084931506849	0.053747572183	0.058852668688
2 m	0.169863013699	0.053853326945	0.059102331682
3 m	0.257534246575	0.053884679239	0.059749598503
4 m	0.336986301370	0.053881404413	0.060144049293
5 m	0.419178082192	0.053879814239	0.060394748240
6 m	0.506849315068	0.053865788152	0.060693480038
9 m	0.756164383562	0.053790318178	0.061347010849
12 m	1.005479452055	0.053631000310	0.061761341974
2 y	2.005479452055	0.053302847382	0.061944569026

The market prices of the DNT options are given in tables 2(i) and 2(ii) of Carr and Crosby (2008) and also repeated in table OLS 7 below.

The vanilla options used in the calibration are as follows:

The strikes of the options were:

Table OLS 4

Type	Put	Put	Call	Call	Call
Delta	-0.1	-0.25	0.5	0.25	0.1
1 w	1.985676533	1.995746582	2.005825905	2.016146155	2.026782346
1 m	1.964798188	1.984991007	2.005165985	2.026551495	2.048947169
2 m	1.946454190	1.975712832	2.004129311	2.034670424	2.067277986
3 m	1.930306722	1.967525291	2.002649845	2.041318966	2.083263932
4 m	1.917032150	1.960754673	2.001145029	2.046622792	2.096240105
5 m	1.904655159	1.954418486	1.999509331	2.051353718	2.107893191
6 m	1.892026677	1.947889457	1.997538603	2.055839695	2.119281951
9 m	1.861407015	1.931790535	1.991109096	2.065527644	2.146374891
12 m	1.836213596	1.918225113	1.983665706	2.071946770	2.168262120
2 y	1.769473956	1.882629161	1.950714348	2.080331789	2.219419900

The market implied volatilities (expressed as percentages) were:

Table OLS 5

Type	Put	Put	Call	Call	Call
Delta	-0.1	-0.25	0.5	0.25	0.1
1 w	5.700	5.413	5.275	5.488	5.850
1 m	5.487	5.225	5.125	5.375	5.763
2 m	5.612	5.300	5.200	5.400	5.837
3 m	5.788	5.425	5.300	5.525	6.013
4 m	5.948	5.552	5.413	5.656	6.168
5 m	6.081	5.662	5.522	5.757	6.272
6 m	6.225	5.787	5.650	5.862	6.375
9 m	6.477	6.015	5.872	6.055	6.572
12 m	6.600	6.138	5.975	6.162	6.700
2 y	6.565	6.145	6.065	6.145	6.655

From the Black and Scholes (1973) formula, this gave market prices of (note that we have multiplied all prices by one million to make the table easier to read):

Table OLS 6

Type	Put	Put	Call	Call	Call
Delta	-0.1	-0.25	0.5	0.25	0.1
1 w	752.3009	2252.1448	5823.1789	2266.1469	766.9851
1 m	1530.3637	4598.3864	11875.4951	4657.6913	1585.5872
2 m	2222.4919	6629.3069	17020.2802	6606.4862	2266.6655
3 m	2832.9392	8393.9460	21352.9725	8313.1579	2870.5892
4 m	3340.8002	9865.8406	24946.0823	9726.8199	3364.3077
5 m	3821.1289	11265.4312	28388.8179	11035.4839	3811.8568
6 m	4315.1858	12713.3665	31952.0847	12350.1934	4256.7381
9 m	5530.2809	16317.4677	40657.1945	15575.0550	5351.0844
12 m	6547.5362	19395.8579	47886.5675	18287.0648	6284.8879
2 y	9431.9065	28404.1025	70181.2591	25957.5749	8831.0148

The first stage in the calibration is to calibrate our model to the market prices of DNT options in order to obtain the parameters of the process for the spot FX rate before the first exit time from the corridor. The results of this calibration for specification 2(ii) (see section 5.2) were:

Table OLS 7

Double-no-touch (DNT) option prices
Cable
06/07/2008

Spot fx rate 2.0060

	Model price	Mid-market	MC price	s/e
Maturity				
	Barrier levels 1.95 / 2.05			
1 m	0.7515594	0.765	0.75143	0.00021
2 m	0.4838907	0.5	0.48412	0.00025
3 m	0.3151016	0.325	0.31538	0.00023
4 m	0.2199347	0.22	0.21984	0.00020
5 m	0.1555874	0.15	0.15581	0.00018
6 m	0.1101323	0.1	0.11024	0.00015
9 m	0.0449567	0.05	0.04493	0.00010
12 m	0.0195439	0.03	0.01963	0.00007
	Barrier levels 1.97 / 2.04			
1 m	0.5115356	0.515	0.51158	0.00025
3 m	0.1070724	0.115	0.10715	0.00015
	Barrier levels 1.98 / 2.03			
1 w	0.8450553	0.85	0.84521	0.00018
1 m	0.2451389	0.245	0.24513	0.00021

Note that table OLS 7 partially repeats table 2(ii) of Carr and Crosby (2008). Just for comparison (clearly, this was not actually necessary for the calibration), we have also included in table OLS 7 the prices (labelled MC price) obtained by Monte Carlo simulation with four million runs as well as the corresponding standard errors (labelled s/e) using the estimated parameters. Again, we see excellent agreement between the prices obtained from equations (A.21) to (A.24) and those obtained by Monte Carlo simulation. For all the prices in table OLS 7, the number of terms (nTerms) used in the Gaver-Stehfest algorithm was set equal to 16. Using equations (A.21) to (A.24), it took approximately 12 seconds to evaluate all twelve option prices in table OLS 7 or an average of approximately one second per option.

The parameters (for specification 2(ii)) obtained from the calibration to DNT options were as follows:

The reciprocals of the mean jump sizes were $b_1 = 66.14146679$, $b_2 = 33.07073340$, $b_3 = 33.07073340$, $b_4 = 66.14146679$. Furthermore the jump intensity rates were $a_1(1) = 0.86961841$, $a_2(1) = 0.85065812$, $a_3(1) = 0.85582605$, $a_4(1) = 0.86438684$, $a_1(2) = 0.32205271$, $a_2(2) = 0.31503099$, $a_3(2) = 0.31694488$, $a_4(2) = 0.32011526$. The diffusion volatilities were $\sigma(1) = 0.04773778$, $\sigma(2) = 0.02905102$. Note that we assumed that the initial state of the Markov chain was 1. The Markov chain transition rates were $\varepsilon_{12} = \varepsilon_{21} = 1.31417031$.

Fixing these parameters, we then priced double barrier knockout options with strikes as given in table OLS 4. The results were as in table OLS 8 (note that we have multiplied all prices by one million to make the table easier to read). The ordering of the columns in table OLS 8 to OLS 13 is the same as in tables OLS 4 to OLS 6 (i.e. put delta -0.1, put delta -0.25, call delta 0.5, call delta 0.25 and call delta 0.1). Just for comparison (clearly, this was not actually necessary for the calibration), we have also included in table OLS 9 the prices obtained by Monte Carlo simulation with four million runs as well as the corresponding standard errors in table OLS 10 using the estimated parameters above. Again, we see excellent agreement between the prices obtained from equations (A.21) to (A.24) and those obtained by Monte Carlo simulation. For all the prices in table OLS 8, the number of terms (nTerms) used in the Gaver-Stehfest algorithm was set equal to 16. It took an average of approximately one second per option to evaluate the option prices in table OLS 8.

Table OLS 8

Double barrier knockout option prices (multiplied by one million)

1 w	472.1306	1853.3481	5136.5846	1639.9943	332.4955
1 m	153.9223	2035.1136	5387.7470	811.3274	0.0742
2 m	0	674.2214	3427.9490	139.2014	0
3 m	0	154.0905	2344.4715	15.7709	0
4 m	0	26.4482	1751.2083	0.6311	0
5 m	0	1.3783	1339.4609	0	0
6 m	0	0	1042.1217	0	0
9 m	0	0	572.9640	0	0
12 m	0	0	340.4993	0	0
2 y	0	0	35.7675	0	0

Table OLS 9

Double barrier knockout option prices (multiplied by one million) obtained by Monte Carlo simulation with four million runs

1 w	474.4089	1856.3676	5129.1120	1638.5540	331.6955
1 m	154.1466	2035.6758	5384.8188	809.7995	0.0781
2 m	0	673.4310	3423.8879	139.2181	0
3 m	0	153.7491	2341.9602	15.9442	0
4 m	0	26.4938	1753.3806	0.6483	0
5 m	0	1.3472	1341.0025	0	0
6 m	0	0	1043.2409	0	0
9 m	0	0	572.4432	0	0
12 m	0	0	339.8398	0	0
2 y	0	0	36.1713	0	0

Table OLS 10
Standard errors (multiplied by one million) of the Monte Carlo prices

1 w	1.2056	2.4211	3.8684	2.1535	0.8782
1 m	0.5167	2.7753	4.8155	1.4649	0.0023
2 m	0	1.4126	4.0901	0.4912	0
3 m	0	0.5566	3.5157	0.1221	0
4 m	0	0.1769	3.1227	0.0137	0
5 m	0	0.0233	2.7901	0	0
6 m	0	0	2.5160	0	0
9 m	0	0	1.9778	0	0
12 m	0	0	1.6115	0	0
2 y	0	0	0.6624	0	0

The next stage in the calibration is to subtract the double barrier knockout option prices in table OLS 8 from the corresponding market vanilla option prices in table OLS 6. This gives us the market implied double barrier knock-in prices. These are as in table OLS 11 (note that we have multiplied all prices by one million to make the table easier to read).

Table OLS 11
Market implied double barrier knock-in option prices (multiplied by one million)

1 w	280.1703	398.7967	686.5943	626.1526	434.4896
1 m	1376.4414	2563.2729	6487.7481	3846.3639	1585.5130
2 m	2222.4919	5955.0856	13592.3312	6467.2848	2266.6655
3 m	2832.9392	8239.8555	19008.5010	8297.3870	2870.5892
4 m	3340.8002	9839.3924	23194.8740	9726.1888	3364.3077
5 m	3821.1289	11264.0530	27049.3570	11035.4839	3811.8568
6 m	4315.1858	12713.3665	30909.9629	12350.1934	4256.7381
9 m	5530.2809	16317.4677	40084.2304	15575.0550	5351.0844
12 m	6547.5362	19395.8579	47546.0682	18287.0648	6284.8879
2 y	9431.9065	28404.1025	70145.4916	25957.5749	8831.0148

In our calibration to vanilla options, we only used the options with maturities of six months (6 m), nine months (9 m), one year (12 m) and two years (2 y). We could have used shorter-dated options in the calibration but we surmised that the prices of very short-dated options would have only a relatively small sensitivity to the parameters of the process of the spot FX rate after the first exit time from the corridor.

We now focus on the calibration to vanilla prices when the dynamics of the spot FX rate after the first exit time from the corridor are those of specification 2(1) (see section 5.2 of Carr and Crosby (2008)). Our calibration fits the model double barrier knock-in option prices (using equation (4.6) and then Laplace inversion) to the market implied double barrier knock-in option prices in table OLS 11.

The parameters (for specification 2(1)) obtained from the calibration to double barrier knock-in option prices were as follows:
The reciprocals of the mean jump sizes were $b_1 = 66.77927761$, $b_2 = 33.38963880$, $b_3 = 33.38963880$, $b_4 = 66.77927761$. Furthermore the jump intensity rates were $a_1(1) = 0.39267742$, $a_2(1) = 0.22509495$, $a_3(1) = 0.26777655$, $a_4(1) = 0.40252010$, $a_1(2) = 1.42029104$, $a_2(2) = 0.81415517$, $a_3(2) = 0.96853198$, $a_4(2) = 1.45589147$. The diffusion volatilities were $\sigma(1) = 0.02638070$, $\sigma(2) = 0.05017149$. The Markov chain started off in state 1 at the instant after the first exit time from the corridor i.e. $\Psi(\tau) = 1$. The Markov chain transition rates were $\varepsilon_{12} = \varepsilon_{21} = 1.44688761$.

With the parameters estimated above, we now display the model implied double barrier knock-in option prices. For comparison, we report the values obtained for the double barrier knock-in option prices obtained by numerical integration (as in appendix 1 of Carr and Crosby (2008), using values of nTerms, numPoints and convergenceTol of 14, 18 and 1.0E-12 respectively) in table OLS 12 as well as those obtained using our analytical results for the CEE2 process (see remark A.7 of Carr and Crosby (2008), with the value of nTerms set equal to 14) in table OLS 13. Again, we see excellent agreement between the two different approaches. Note that in tables OLS 12 and OLS 13, we have multiplied all prices by one million to make the tables easier to read.

Table OLS 12
Model implied double barrier knock-in option prices (multiplied by one million)

6 m	4677.8895	13391.0202	32457.7954	12627.4017	4457.0058
9 m	5243.0114	15866.0357	40001.0092	15007.4664	4994.3079
12 m	5938.8728	18566.8671	47025.8270	17452.7964	5560.0836
2 y	9196.6631	29349.2253	71389.1719	27024.3794	8358.3237

Table OLS 13
Model implied double barrier knock-in option prices (multiplied by one million)

6 m	4677.8663	13391.0677	32457.8034	12627.4118	4456.9868
9 m	5242.9755	15866.0665	40001.0634	15007.4472	4994.2518
12 m	5938.8746	18566.7952	47025.8289	17452.7135	5559.9852
2 y	9196.6831	29349.1883	71388.9927	27024.5226	8358.2214

It took approximately 82 seconds to evaluate all twenty option prices in table OLS 12 and approximately 34 seconds to evaluate all twenty option prices in table OLS 13. The latter equates to an average of approximately 1.7 seconds per option.

If we add the model implied double barrier knock-in prices in table OLS 13 (or table OLS 12) to the corresponding double barrier knockout option prices in table OLS 8, we get the model implied vanilla option prices.

We display the model implied vanilla option prices (based on table OLS13 i.e. with the results obtained using our analytical results for the CEE2 process (see remark A.7 of Carr and Crosby (2008))) in table OLS14 below (note that we have multiplied all prices by one million to make the table easier to read).

Table OLS 14
Model implied vanilla option prices (multiplied by one million)

6 m	4677.8663	13391.0677	33499.9251	12627.4118	4456.9868
9 m	5242.9755	15866.0665	40574.0274	15007.4472	4994.2518
12 m	5938.8746	18566.7952	47366.3282	17452.7135	5559.9852
2 y	9196.6831	29349.1883	71424.7601	27024.5226	8358.2214

We can then convert these vanilla option prices to implied volatilities. These are displayed (as percentages) in graphical form in figures 5 to 8 of Carr and Crosby (2008). For convenience, we have also displayed these implied volatilities (expressed as percentages) in table OLS 15. Table OLS 15 is read as follows: We display results for six months (6 m), nine months (9 m), twelve months (12 m) and two years (2 y) separately. In each sub-table, the first row shows market implied volatilities, the second row displays model implied volatilities for specification 2(1) (which is the specification we have focussed on), the third row displays model implied volatilities for specification 2(2) (which we have not described in detail in this on-line supplement but which is described in section 5.2 of Carr and Crosby (2008)) and the fourth row displays implied volatilities in the first experiment described in section 5.2 of Carr and Crosby (2008) (these latter results are labelled "Using parameters implied from DNT options" in figures 5 to 8 of Carr and Crosby (2008)).

Table OLS 15

6 m					
Market	6.225	5.787	5.650	5.862	6.375
Specification 2(1)	6.368	5.938	5.930	5.924	6.455
Specification 2(2)	6.474	5.981	5.979	5.949	6.437
First experiment	7.295	6.882	6.760	6.954	7.451

9 m					
Market	6.477	6.015	5.872	6.055	6.572
Specification 2(1)	6.381	5.932	5.859	5.950	6.452
Specification 2(2)	6.362	5.978	5.901	6.002	6.507
First experiment	7.184	6.819	6.716	6.889	7.326
12 m					
Market	6.600	6.138	5.975	6.162	6.700
Specification 2(1)	6.421	6.004	5.906	6.027	6.485
Specification 2(2)	6.300	6.001	5.933	6.089	6.561
First experiment	7.082	6.760	6.673	6.826	7.219
2 y					
Market	6.565	6.145	6.065	6.145	6.655
Specification 2(1)	6.516	6.256	6.191	6.270	6.556
Specification 2(2)	6.215	6.105	6.111	6.271	6.585
First experiment	6.820	6.623	6.583	6.673	6.931

Although we have focussed on describing the calibration to the market data as of 6th July 2007 for specifications 2(ii) and 2(1) (see section 5.2 of Carr and Crosby (2008)), the other calibrations performed in Carr and Crosby (2008) are conceptually identical. In the next section, we provide some more information on the calibration to market data as of 31st May 2007.

OLS 5. Calibration to market data as of 31st May 2007

In this section, we describe in more detail the calibration to market data for cable (USD/STG) as of 31st May 2007 and provide the market data used.

The market data as of 31st May 2007 used for the calibration was as follows: The (initial) spot FX rate (number of USD per STG) was $S(t_0) = 1.97575$ (mid-market). The corridor levels were $L = 1.9200$ and $U = 2.0200$. The interest-rates (continuously compounded) were as in table OLS 16.

Table OLS 16

Tenor	Time in years	USD interest-rates	STG interest-rates
1 w	0.019178082192	0.053648747288	0.056509949846
1 m	0.095890410959	0.053851954447	0.056996176928
6 w	0.115068493151	0.053873385530	0.057193863013
2 m	0.183561643836	0.053912247036	0.057562999027
3 m	0.263013698630	0.053932167928	0.057736087704
4 m	0.345205479452	0.053882573985	0.057955672608
5 m	0.432876712329	0.053798924507	0.058384711938
6 m	0.512328767123	0.053747845764	0.058640473088
9 m	0.761643835616	0.053462207085	0.059253418899
12 m	1.013698630137	0.053096558316	0.059594729261
2 y	2.000000000000	0.052042830890	0.059746195171

The first stage in the calibration was to calibrate our CEE2 process (equation (2.1) of Carr and Crosby (2008)) with $M = 2$ and the intensity rates and the diffusion volatility assumed to be constants (which is the same as the Kou (2002) DEJD model) to the market prices of DNT options.

These market prices are in table 1 of Carr and Crosby (2008) so we do not reproduce them here. The parameters obtained were as follows: The reciprocals of the mean jump sizes were $b_1 = 50.22489527$, $b_2 = 49.69270908$. Furthermore the jump intensity rates were $a_1(1) = 1.61516621$, $a_2(1) = 1.65767492$. The diffusion volatility was $\sigma(1) = 0.04417247$. Note that there is effectively only one state in the Markov chain. The model prices of the DNT options are shown in table 1 of Carr and Crosby (2008) so we do not reproduce them here.

The second stage in the calibration was to calibrate to vanilla options as described in section 5 of Carr and Crosby (2008) for each of the three specifications. The parameters obtained were as follows:

For specification (1):
The reciprocals of the mean jump sizes were $b_1 = 51.53354972$, $b_2 = 50.51600129$. Furthermore the jump intensity rates were $a_1(1) = 1.63863122$, $a_2(1) = 1.62232660$. The diffusion volatility was $\sigma(1) = 0.03072450$.

For specification (2):
$C = 1.44408557$, $G = 37.28521320$, $M = 35.06325613$, $Y = 0.12754093$.
The volatility of the Brownian motion component was 0.01264190.

For specification (3):
$C_1^A = C_2^A = 1.68153893$, $C_1^N = C_2^N = 1.18488618$,
$G = 36.12737962$, $M = 37.17746004$, $Y = 0.12725951$,
$\lambda_1^{NA} = \lambda_2^{NA} = 1.11231888$, $\lambda_1^{AN} = \lambda_2^{AN} = 2.48734859$. The volatility of the Brownian motion component was 0.01097453.

The comparison between the market implied volatilities and the model implied volatilities for the three different specifications (as well as the model implied volatilities for the first experiment) are shown (as percentages) in graphical form in figures 1 to 4 of Carr and Crosby (2008). For convenience, we have also displayed these implied volatilities (expressed as percentages) in table OLS 17. Table OLS 17 is read as follows: We display results for six months (6 m), nine months (9 m), twelve months (12 m) and two years (2 y) separately. In each sub-table, the first row shows market implied volatilities, the second, third and fourth rows display model implied volatilities for specifications 1, 2 and 3 respectively and the fifth row displays implied volatilities in the first experiment described in section 5.1 of Carr and Crosby (2008) (these latter results are labelled "Using parameters implied from DNT options" in figures 1 to 4 of Carr and Crosby (2008)).

Table OLS 17

6 m					
Market	6.350	5.912	5.700	5.887	6.250
Specification (1)	6.497	6.097	6.073	6.054	6.383
Specification (2)	6.472	6.060	6.051	6.058	6.480
Specification (3)	6.452	6.031	6.017	6.026	6.440
First experiment	6.945	6.595	6.486	6.585	6.867

9 m					
Market	6.559	6.100	5.876	6.073	6.472
Specification (1)	6.449	6.049	5.985	6.020	6.303
Specification (2)	6.419	6.023	5.999	6.059	6.414
Specification (3)	6.436	6.013	5.980	6.026	6.361
First experiment	6.968	6.656	6.565	6.641	6.861

12 m					
Market	6.500	6.063	5.825	6.038	6.500
Specification (1)	6.405	6.019	5.936	5.989	6.228
Specification (2)	6.375	5.996	5.948	6.041	6.344
Specification (3)	6.421	6.008	5.938	6.008	6.288
First experiment	6.971	6.686	6.609	6.667	6.843
2 y					
Market	6.440	6.015	5.925	6.080	6.580
Specification (1)	6.341	5.978	5.882	5.930	6.100
Specification (2)	6.318	5.962	5.899	5.991	6.219
Specification (3)	6.417	6.018	5.915	5.973	6.166
First experiment	6.988	6.750	6.723	6.733	6.823

For brevity, we have not displayed the strikes of the vanilla options used in the second stage of the calibration but these are easily computed by using the market implied volatilities in table OLS 17 and then solving for the strike which gives the appropriate delta. The market prices can then be obtained from the Black and Scholes (1973) formula.

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