Pricing Commodity Hybrid Derivatives

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Modelling commodity prices with a multi-factor jump-diffusion model

This presentation partially draws on my papers "**A multi-factor jump-diffusion model for Commodities**", "**Pricing a class of exotic commodity options in a multi-factor jump-diffusion model**" and "**Valuing Inflation Futures Contracts**" (all submitted for publication) as well as "**Commodity options optimized**" (Risk Magazine, May 2006 p72-77).

Talk outline

- Many banks have (or would like to have) generic multi-asset Monte Carlo engines which are able to simulate any number of assets, of possibly different types (eg commodities, inflation), in possibly different currencies.
- I will outline how this can be done in a simple and generic way focussing mostly on commodities and inflation and just briefly outlining interest-rates and fx.
- I will also price a particular commodity/inflation hybrid derivative using Fourier methods.

Stochastic interest-rates

- We will assume interest-rates are stochastic throughout.
- Since including stochastic interest-rates is usually the trickier part, I will start by defining the notation and the model for stochastic interest-rates.
- I will work with a one-factor Gaussian HJM interest-rate model but everything generalises to a multi-factor version without further ado.

Stochastic Interest-rates

- We denote the (continuously compounded) risk-free short rate, at time t , by $r(t)$ and we denote the price of a zero coupon bond, at time t maturing at time T by $P(t,T)$. We assume that bond prices follow the extended Vasicek (1-f Gaussian HJM) process, namely,

$$\frac{dP(t,T)}{P(t,T)} = r(t)\,dt + \sigma_P(t,T)\,dz_P(t)$$

$$\sigma_P(t,T) \equiv \frac{\sigma_r}{\alpha_r}\left(1-\exp\left(-\alpha_r(T-t)\right)\right)$$

- where σ_r and α_r are positive constants.
- Define the (Gaussian) state variables

$$X_P(t) = \int_{t_0}^{t} \sigma_r \exp\left(-\alpha_r (t-s)\right) dz_P(s)$$

$$Y_P(t) = \int_{t_0}^{t} \sigma_r dz_P(s)$$

where t_0 is the initial time

ETS can write $r(t)$ and $P(t,T)$ in terms of $X_P(t)$

Specifically :

$$P(t,T)=\frac{P(t_0,T)}{P(t_0,t)}\ \exp\left(\frac{1}{2}\int_{t_0}^{t}\left[\sigma_P^2(s,t)-\sigma_P^2(s,T)\right]ds\right)$$

$$\exp\left(\frac{1}{\alpha_r}\left[1-\exp(-\alpha_r(T-t))\right]X_P(t)\right)$$

Note $X_P(t)$ is easy to simulate without discretization error and hence also simulating bond prices is straightforward.

Pricing interest-rate derivatives

- In essence, the bond price formula gives us all but one component of what we need to price IR derivatives by simulation (at least for one-factor interest-rate model – the extension to a multi-factor Gaussian model is extremely straightforward).
- The last remaining component we need when interest-rates are stochastic is the stochastic discounting term ie the reciprocal of the money market account numeraire:

Simulating stochastic discounting term

- Need expected *discounted* payoffs:
- But can simulate $\exp\left(-\int_{t_0}^{t} r(s)ds\right)$ via:

$$\exp\left(-\int_{t_0}^{t} r(s)ds\right) = P(t_0,t)\exp\left(-\frac{1}{2}\int_{t_0}^{t} \sigma_P^2(s,t)ds\right)\exp\left(\frac{1}{\alpha_{rk}}\left[Y_P(t) - X_P(t)\right]\right)$$

- Note: Have to simulate $Y_P(t)$ and $X_P(t)$
- Equivalent to simulating in forward measure without actually doing so.

Empirically observed features captured by the Crosby (2005) model

- Spot commodity prices exhibit mean reversion.
- Futures (and forward) commodity prices have instantaneous volatilities which usually (but not absolutely always) decline with increasing tenor.
- Jumps are somewhat more common and certainly much larger in magnitude than in other markets (eg equities or fx).
- A common feature in commodities (esp. Gas and Electricity) is that when there is a jump, the spot and short-dated futures (or forward) prices jump by a large amount but long-dated contracts hardly jump at all (to our knowledge no previous models had accounted for this feature).
- Convenience yields are usually highly volatile.

The model

The Crosby (2005) model is a no-arbitrage model which automatically fits the initial term structure of futures (or forward) commodity prices.

- Let us denote the futures commodity price at time t for delivery at time T by $H(t,T)$

- We take as given our initial (ie at time t_0) term structure of futures commodity prices.
- Futures prices are martingales under the EMM. (Cox et al. (1981)).

The model

- Then we assume that the dynamics of futures commodity prices in the EMM are:

$$\frac{dH(t,T)}{H(t,T)} = \sum_{k=1}^{K} \sigma_{Hk}(t,T)\, dz_{Hk}(t) - \sigma_P(t,T)\, dz_P(t)$$

$$+ \sum_{m=1}^{M} \left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right) dN_{mt}$$

$$- \sum_{m=1}^{M} \lambda_m(t) E_{Nmt} \left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right) dt$$

- K is the number of Brownian factors (for example, 1, 2 or 3).
- We assume that the volatility functions $\sigma_{Hk}(t,T)$ are deterministic.
- M is the number of Poisson processes.

- For each k, $k=1,2,...,K$, $dz_{Hk}(t)$ denotes standard Brownian increments. We denote the correlation (assumed constant) between $dz_P(t)$ and $dz_{Hk}(t)$ by ρ_{PHk}, for each k, and the correlation (assumed constant) between $dz_{Hk}(t)$ and $dz_{Hj}(t)$ by ρ_{HkHj} for each k and j.
- $\rho_{HkHj}=1$ if $k=j$

Jump processes

- For each m , $m = 1,\ldots,M$, $\lambda_m(t)$ are the (assumed) deterministic intensity rates of the M Poisson processes.
- $b_m(u)$ for each m are non-negative deterministic functions. We call these the jump decay coefficient functions.
- γ_{mt} are termed the spot jump amplitudes.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.1 in the paper:
- The spot jump amplitudes are (known) constants. In this case, the jump decay coefficient functions $b_m(u)$ can be non-negative (but otherwise arbitrary) deterministic functions.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.2 in the paper:
- The spot jump amplitudes are assumed to be independent and identically distributed random variables (assumed independent of everything else). In this case, the jump decay coefficient functions must be equal to zero. ie $b_m(t) \equiv 0$ for all t

- It is convenient to define:

$$e_m(t,T) \equiv \sum_{m=1}^{M} \lambda_m(t) E_{Nmt}\left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right)$$

- This is a deterministic quantity.

- Then by Ito: $d\left(\ln H(t,T)\right) =$

$$-\frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^{2}(t,T)+\sigma_{P}^{2}(t,T)-2\sum_{k=1}^{K}\rho_{PHk}\sigma_{P}(t,T)\sigma_{Hk}(t,T)\right\}dt$$

$$-\frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}2\rho_{HkHj}\sigma_{Hk}(t,T)\sigma_{Hj}(t,T)\right\}dt$$

$$+\sum_{k=1}^{K}\sigma_{Hk}(t,T)dz_{Hk}(t)-\sigma_{P}(t,T)dz_{P}(t)$$

$$+\sum_{m=1}^{M}\gamma_{mt}\exp\left(-\int_{t}^{T}b_{m}(u)du\right)dN_{mt}-\sum_{m=1}^{M}e_{m}(t,T)dt$$

Implications

- For Crude oil, petroleum products and especially **Natural Gas, Electricity: Short end of futures curve jumps a lot, long end hardly jumps at all (existing models do not seem to have this).**
- For Gold: Jumps are less of a feature (but they do happen).
- “Gold trades somewhat like a currency”.
- ie jumps cause parallel shift in futures (and forward) curve.

- In general, $C_t \equiv H(t,t)$ would be non-Markovian but we would like it to be a Markov process in a finite number of state variables.
- We consider the functional form for the volatilities:

$$\sigma_{Hk}(s,t) = \eta_{Hk}(s) + \chi_{Hk}(s)\exp\left(-\int_s^t a_{Hk}(u)\,du\right)$$

where $\eta_{Hk}(s)$, $\chi_{Hk}(s)$ and $a_{Hk}(u)$

are deterministic functions.

Why?

Gaussian state variables:

- Define the Gaussian state variables, for each $k,\ k = 1,2,\ldots,K$:

$$X_{Hk}(t) = \int_{t_0}^{t} \exp\left(-\int_{t_0}^{t} a_{Hk}(u)\,du\right) \chi_{Hk}(s) \exp\left(-\int_{s}^{t_0} a_{Hk}(u)\,du\right) dz_{Hk}(s)$$

$$Y_{Hk}(t) = \int_{t_0}^{t} \eta_{Hk}(s)\, dz_{Hk}(s)$$

Poisson state variables

- Define the Poisson jump state variable (for each m) :

$$X_{Nm}(t) = \int_{t_0}^{t} \gamma_{ms} \exp\left(-\int_{s}^{t} b_m(u) du \right) dN_{ms}$$

- Then with some algebra….

- We have the following expression for the futures commodity price $H(t,T)$ at time t to time T in terms of the initial (ie at time t_0) futures commodity price and the state variables:

$$
\begin{aligned}
H(t,T) = H(t_0,T)\exp&\left(\int_{t_0}^{t} -\frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^2(s,T)+\sigma_P^2(s,T)\right\}ds\right) \\
\exp&\left(\int_{t_0}^{t} -\frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}2\rho_{HkHj}\sigma_{Hk}(s,T)\sigma_{Hj}(s,T)-\sum_{k=1}^{K}2\rho_{PHk}\sigma_P(s,T)\sigma_{Hk}(s,T)\right\}ds\right) \\
\exp&\left(\frac{\exp(-\alpha_r(T-t))}{\alpha_r}X_P(t)-\frac{1}{\alpha_r}Y_P(t)\right) \\
\exp&\left(\sum_{k=1}^{K}Y_{Hk}(t)+\sum_{k=1}^{K}\left[\exp\left(-\int_{t}^{T}a_{Hk}(u)du\right)X_{Hk}(t)\right]\right) \\
\exp&\left(\sum_{m=1}^{M}\left(\left\{\exp\left(-\int_{t}^{T}b_m(u)du\right)X_{Nm}(t)\right\}-\int_{t_0}^{t}e_m(s,T)ds\right)\right)
\end{aligned}
$$

Monte Carlo Simulation

- Therefore, we can easily do Monte Carlo simulation if we can simulate the state variables:
- Gaussian state variables are straightforward (and note that they are of the same mathematical form as the state variables we defined to simulate stochastic bond prices and the stochastic discounting term).
- We show, in the paper, that we can also simulate the Poisson state variables (and without discretization error bias).

Pricing standard European options

- We can calibrate our model to the prices of standard European options in the market because we can price standard European options.
- There are two methods to do this (see papers):
- MCIATJ (Monte Carlo integration over the arrival times of the jumps)
- Or: (see Risk Magazine, May 2006) FTPS (a Fourier method which, in general, relies on a power series expansion of terms appearing in the characteristic function). Usually FTPS is faster.

Inflation

- We construct our model by using the "cross-currency model" analogy.
- We use subscript re to denote variables related to the term structure of real interest-rates.
- Therefore, $r_{re}(t)$ denotes the real short rate at time t and $P_{re}(t,T)$ denotes the real interest-rate discount factor, at time t, to time T. Also, we denote the spot inflation (CPI) index level, at time t, by $I(t)$.

Standard 3-factor model for inflation

$$\frac{dP_{re}(t,T)}{P_{re}(t,T)} = \left(r_{re}(t) - \rho_{\mathrm{Pr}eI}\sigma_I(t)\sigma_P^{re}(t,T)\right)dt + \sigma_P^{re}(t,T)dz_{\mathrm{Pr}e}(t)$$

where $\rho_{\mathrm{Pr}eI}$ is the assumed constant correlation between the CPI index and real discount factors.

Further assume: $$\frac{dI(t)}{I(t)} = \left(r(t) - r_{re}(t)\right)dt + \sigma_I(t)dz_I(t)$$

vol. of CPI index assumed deterministic and volatility term $\sigma_P^{re}(t,T)$ assumed deterministic and of the extended Vasicek form.

- Denote the forward CPI index level, at time t to time T by $F_I(t,T)$, defined via

$$F_I(t,T) = \frac{I(t)P_{re}(t,T)}{P(t,T)}$$ Then Ito =>

$$\frac{dF_I(t,T)}{F_I(t,T)} = \{\mathrm{cov}(\sigma_P(t,T)dz_P(t),$$

$$\sigma_P(t,T)dz_P(t) - \sigma_I(t)dz_I(t) - \sigma_P^{re}(t,T)dz_{\mathrm{P}re}(t)\}dt$$

$$+\sigma_I(t)dz_I(t) + \sigma_P^{re}(t,T)dz_{\mathrm{P}re}(t) - \sigma_P(t,T)dz_P(t)$$

- Notice that $F_I(t,T)$ is log-normally distributed (**drift term and volatility term** in the SDE are **deterministic**).
- Now also define state variables:

$$Y_I(t)=\int_{t_0}^{t}\sigma_I(s)dz_I(s)$$

$$X_P^{re}(t)=\int_{t_0}^{t}\sigma_r^{re}\exp\left(-\alpha_r^{re}(t-s)\right)dz_{\mathrm{Pre}}(s)$$

$$Y_P^{re}(t)=\int_{t_0}^{t}\sigma_r^{re}(s)dz_{\mathrm{Pre}}(s)$$

Forward CPI in terms of the state variables

- Then:

$$F_I(t,T) = F_I(t_0,T)\exp\left(\int_{t_0}^{t}\left(\frac{1}{2}\sigma_P^2(s,T) - \frac{1}{2}\sigma_I^2(s) - \frac{1}{2}\sigma_P^{re^2}(s,T)\right)ds\right.$$

$$\left. - \int_{t_0}^{t}\rho_{P\mathrm{Re}I}\,\sigma_I(s)\sigma_P^{re}(s,T)ds\right)$$

$$\exp\left(Y_I(t) + \frac{1}{\alpha_r^{re}}Y_P^{re}(t) - \frac{1}{\alpha_r^{re}}\exp\left(-\alpha_r^{re}(T-t)\right)X_P^{re}(t)\right)$$

$$\exp\left(-\frac{1}{\alpha_r}Y_P(t) + \frac{1}{\alpha_r}\exp\left(-\alpha_r(T-t)\right)X_P(t)\right)$$

- This last equation means we can easily simulate the forward CPI $F_I(t,T)$. Then we can easily obtain $I(t)$ via $I(t)=F_I(t,t)$

  Note the extra ingredients:

  To simulate (nominal) bond prices (as we mentioned earlier in the talk) we only have to simulate $X_P(t)$

whereas to simulate the forward CPI $F_I(t,T)$

we must simulate all of $Y_I(t), Y_P^{re}(t), X_P^{re}(t), \; Y_P(t), X_P(t)$

- We now introduce a quantity $H_I(t,T)$ defined as follows:

$$H_I(t,T) = \frac{I(t)P_{re}(t,T)}{P(t,T)} \exp\left(\int_t^T \{ \mathrm{cov}(\sigma_P(s,T)dz_P(s), \right.$$

$$\left. \sigma_P(s,T)dz_P(s) - \sigma_I(s)dz_I(s) - \sigma_P^{re}(s,T)dz_{\mathrm{P}re}(s) \} ds \right)$$

$$= F_I(t,T) \exp\left(\int_t^T \{ \mathrm{cov}(\sigma_P(s,T)dz_P(s), \right.$$

$$\left. \sigma_P(s,T)dz_P(s) - \sigma_I(s)dz_I(s) - \sigma_P^{re}(s,T)dz_{\mathrm{P}re}(s) \} ds \right)$$

Ito => $$\frac{dH_I(t,T)}{H_I(t,T)} = \sigma_I(t)dz_I(t) + \sigma_P^{re}(t,T)dz_{\mathrm{Pre}}(t) - \sigma_P(t,T)dz_P(t)$$

- Hence $H_I(t,T)$ is a martingale.
- Incidentally, $H_I(t,T)$ is mathematically the same as the value, at time t, of a **synthetic** futures contract on CPI with delivery value $I(T)$
- (but this should not be confused with actual CPI futures contracts, traded on exchanges, whose delivery value is the ratio of CPI at expiry time divided by CPI at some earlier time which, as an aside, my paper "Valuing Inflation Futures Contracts" provides formulae for).

Option on real return on a commodity

- Consider a European option whose payoff at time T is:

$$\max\left(\eta\left(\frac{H(T,T) - K^{*}\left[I(T)\right]^{\varepsilon}}{\left[I(T)\right]^{\alpha}} \right), 0 \right)$$

- ε and α (which need not be integers but, in practice, probably would be) give us flexibility on payoff.

  $\eta = +1/-1$ call/put

  K^{*} is a constant multiplier (eg, might be the ratio of the (known) futures commodity price at the time the option was written divided by the (known) spot CPI index level at the time the option was written).

Option on real return on a commodity

- This can be viewed as an option on the real (ie after adjustment for inflation) return of a commodity (for $\alpha = 0, \varepsilon = 1$ or $\alpha = 1, \varepsilon = 1$).
- How can we value this option at time t , where $t \leq T$?
- Rewrite the payoff in terms of $H_I(T,T)$:

$$\max\left(\eta\left(\frac{H(T,T) - K^*\left[H_I(T,T)\right]^{\varepsilon}}{\left[H_I(T,T)\right]^{\alpha}}\right),0\right)$$

- Define for any time $t^* \geq t$

$$Y\left(t^*, T; t\right) \equiv \log\left(\frac{H\left(t^*, T\right)}{\left[H_I\left(t^*, T\right)\right]^{\varepsilon}} \Big/ \frac{H\left(t, T\right)}{\left[H_I\left(t, T\right)\right]^{\varepsilon}} \right)$$

- The price of our option at time t is:

$$E_t\left[\exp\left(-\int_t^T r(s)ds \right) \max\left(\eta\left(\frac{H(T,T) - K^*\left[H_I(T,T)\right]^{\varepsilon}}{\left[H_I(T,T)\right]^{\alpha}} \right), 0 \right) \right]$$

- Which we can write as : $M_1 + M_2 - M_3$

- Where $M_1 \equiv \frac{(1+\eta)}{2} E_t\left[\exp\left(-\int_t^T r(s)ds\right) H(T,T)\left[H_I(T,T)\right]^{-\alpha}\right]$

- And $M_2 \equiv \frac{(1-\eta)}{2} E_t\left[\exp\left(-\int_t^T r(s)ds\right) K^*\left[H_I(T,T)\right]^{\varepsilon-\alpha}\right]$

(can work these out explicitly in our model)

- And where

$$M_3 \equiv E_t\left[\exp\left(-\int_t^T r(s)ds\right)[H_I(T,T)]^{\varepsilon-\alpha}\min\left(\frac{H(t,T)}{[H_I(t,T)]^{\varepsilon}}\exp(Y(T,T;t)), K^*\right)\right]$$

We can compute this with Fourier methods as follows:

- Define

$$f\big(Y(T,T;t)\big) \equiv \min\left(\frac{H(t,T)}{\left[H_I(t,T)\right]^{\varepsilon}} \exp\big(Y(T,T;t)\big), K^* \right)$$

And then write it in terms of its F.T. $\hat{f}(z)$ ie

$$f\big(Y(T,T;t)\big) = \frac{1}{2\pi} \int_{iz_i-\infty}^{iz_i+\infty} \exp\big(-izY(T,T;t)\big)\hat{f}(z)dz$$

Can show

$$\hat{f}(z) = \frac{H(t,T)}{\left[H_I(t,T)\right]^{\varepsilon}} \left(\frac{1}{z^2 - iz} \right) \left(\frac{K^*\left[H_I(t,T)\right]^{\varepsilon}}{H(t,T)} \right)^{iz+1}$$

- Then

$$M_3 \equiv E_t\left[\exp\left(-\int_t^T r(s)ds\right)\left[H_I(T,T)\right]^{\varepsilon-\alpha}\frac{1}{2\pi}\int_{iz_i-\infty}^{iz_i+\infty}\exp(-izY(T,T;t))\hat{f}(z)dz\right]$$

$$=\frac{1}{2\pi}\int_{iz_i-\infty}^{iz_i+\infty}E_t\left[\exp\left(-\int_t^T r(s)ds\right)\left[H_I(T,T)\right]^{\varepsilon-\alpha}\exp(-izY(T,T;t))\right]\hat{f}(z)dz$$

$$\equiv\frac{1}{2\pi}\int_{iz_i-\infty}^{iz_i+\infty}\Phi(-z;t,T)\hat{f}(z)dz$$

where we call $\Phi(-z;t,T)$ the “extended” characteristic function.

"Extended" characteristic function

- Explicitly: $\Phi\left(-z;t,T\right)$

$$= E_t\left[\exp\left(-\int_t^T r(s)ds\right)\left[H_I(T,T)\right]^{\varepsilon-\alpha}\exp\left(-izY(T,T;t)\right)\right]$$

- ie the "extended" characteristic function is the expected value of the product of three random variables

Option price formula

- We have a formula for the option price, at time t :

$$M_1 + M_2 - \frac{1}{2\pi} \int_{iz_i - \infty}^{iz_i + \infty} \Phi(-z; t, T) \hat{f}(z) dz$$

- Need 0 < imag(z) < 1
- This formula is valid for any underlying model for which we can evaluate the “extended” characteristic function.
- So this approach should also be applicable for other models.

- We can work out the “extended” characteristic function that we need using results in Crosby (2006).
- In general, it is not analytic.
- But we can compute it very quickly with the aid of some power series expansions (see papers for details).
- Hence compute our option price with just a single numerical integration (ie to compute the Fourier inversion).

- In Crosby (2006) “Pricing a class of exotic commodity options in a multi-factor jump-diffusion model”, we considered (amongst other things) a much wider class of exotic options than we have considered today including forward start options, cliquet options and spread options. All can be priced with this Fourier methodology.

- F.X. options and cross-currency commodity derivatives

Deterministic terms

- Need to compute lots of terms like

$$\int_{t_0}^{t} \sigma_P^2(s,t)\,ds \qquad \int_{t_0}^{t} \rho_{P\mathrm{Re}I}\,\sigma_I(s)\,\sigma_P^{re}(s,T)\,ds$$

$$\exp\left(\int_{t_0}^{t}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}\rho_{HkHj}\,\sigma_{Hk}(s,T)\,\sigma_{Hj}(s,T)-\sum_{k=1}^{K}\rho_{PHk}\,\sigma_P(s,T)\,\sigma_{Hk}(s,T)\right\}ds\right)$$

- **But: We Never need to work out these terms explicitly.** All deterministic terms (whether for commodities, bonds, fx, inflation) are of the form:

Deterministic terms

$$\int_{t_{start}}^{t_{end}} \left(k_1 + C_1 \exp\left(-\omega_1\left(T_1 - s\right)\right)\right)\left(k_2 + C_2 \exp\left(-\omega_2\left(T_2 - s\right)\right)\right) ds =$$

$$k_1 k_2 \left(t_{end} - t_{start}\right) + \frac{k_1 C_2}{\omega_2}\left\{\exp\left(-\omega_2\left(T_2 - t_{end}\right)\right) - \exp\left(-\omega_2\left(T_2 - t_{start}\right)\right)\right\} +$$

$$\frac{k_2 C_1}{\omega_1}\left\{\exp\left(-\omega_1\left(T_1 - t_{end}\right)\right) - \exp\left(-\omega_1\left(T_1 - t_{start}\right)\right)\right\} +$$

$$\frac{C_1 C_2}{\left(\omega_1 + \omega_2\right)} \exp\left(-\left(\omega_1 T_1 + \omega_2 T_2\right)\right)\left\{\exp\left(\left(\omega_1 + \omega_2\right) t_{end}\right) - \exp\left(\left(\omega_1 + \omega_2\right) t_{start}\right)\right\}$$

- Evaluate this expression in a sub-function and call in “for loops”

Summary

- Assumed bond prices, fx rates, spot CPI index levels are log-normal. Assumed (log) futures commodity prices are Gaussian + jumps.
- Can easily simulate all these prices (and hence also price exotics) by simulating the underlying state variables.
- Mathematically identical Gaussian state variables across all these different asset classes.
- There is no discretisation error bias involved in the simulation.

Summary

- The Crosby (2005) commodity model has following features:
- No-arbitrage model which automatically fits initial futures (or forward) commodity price curve.
- Can show log Spot price exhibits mean reversion.
- Allows for multiple jump processes of different types.
- Long-dated futures commodity prices can jump by less than short-dated futures.
- Generates stochastic convenience yields without further ado.

Copies of the papers

- The papers which I mentioned earlier can be found on the website of the Centre for Financial Research at Cambridge University:

  http://mahd-pc.jbs.cam.ac.uk/seminar/2005-6.html

  or for "**Commodity options optimised**", Risk magazine, May 2006, p72-77.