FX and cross-currency options modelling with Levy processes time-changed by other Levy processes

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Talk outline

- Stochastic Skew model of Carr and Wu.
- Stochastic time-changing by non-Gaussian OU processes (Barndorff-Nielsen and Shephard).
- An alternative specification to generate Stochastic Skew (ie stochastic risk-reversals).
- Pricing barrier options and hybrids.
- Comparison of the different models / specifications.

FX and FX Options markets – stylised empirical observations

- Volatility (both implied and historical) changes randomly, fx options markets imply skews / smiles, jumps in underlying are observed, distribution of fx returns (or log changes) are fat-tailed - but NOT as fat as for equities.
- A key feature in fx options which is NOT observed in other markets is stochastic skew

  ie risk-reversals change in magnitude and actually also change sign on a moderately frequent basis (cf Equity options which are nearly always negatively skewed).

Stochastic Time-changing

- As a primer, it’s well-known that Heston can be thought of as a Brownian motion with a CIR time-change.
- A model consisting of Heston (perhaps, + jumps (ie the Bates model)) can capture the first set of empirically observed features but it cannot capture stochastic skew.
- To capture stochastic skew, Carr and Wu proposed their Stochastic Skew model.

Basic idea of Carr and Wu

- Consider a Levy process which is positively skewed (eg it can only jump up) and apply a stochastic time-change via a CIR process.
- Consider a Levy process which is negatively skewed (eg it can only jump down) and apply a stochastic time-change via a CIR process which is independent of the CIR process for the positively skewed component.
- Add the two together. Assume that these are the dynamics (with an appropriate choice of drift) of log fx rate (under a risk-neutral EMM).
- Because the stochastic clocks on the positively skewed and negatively skewed components can run at different speeds in the future, the model can generate stochastic skew ie the sign and magnitude of risk-reversals can change over time.

Is there an alternative to CIR?

- Stochastic time change by a CIR process has tended to dominate the literature.
- Unfortunately, in general, CIR is not very tractable. Specifically, despite the work of Broadie and Kaya, it is very difficult to Monte Carlo simulate CIR and hence Levy processes time-changed by CIR without discretisation error.

  => tricky to accurately price exotics and get hedge ratios by MC simulation.

  In addition, for a particular combination of parameters (which is very often the case in practice), the stochastic vol. (or stochastic activity rate) hits zero infinitely often.

  This is unrealistic on financial grounds (and the discretisation error becomes especially problematic).

Barndorff-Nielsen and Shephard

- By contrast, Ole Barndorff-Nielsen and Neil Shephard have proposed using non-Gaussian OU processes.
- Basic idea – use an OU process with spectrally positive Levy process (ie it can only have upwards jumps (it must have no Brownian component and must have finite variation)).
- This process is interpreted as the square of instantaneous volatility or the activity rate of a stochastic clock ie a process to induce a stochastic time-change.

Non-Gaussian OU processes

- SDE is: $d\sigma_i^2(t) = -\lambda_i \sigma_i^2 dt + dz_i(\lambda_i t)$

  with $\sigma_i^2(t_0) > 0$ and $t_0 \equiv 0$, $z_i(t_0) \equiv 0$
  where $z_i(t)$ is called the BDLP (Background Driving Levy Process) and λ_i
  is a strictly positive constant. Unusual timing (ie $z_i(\lambda_i t)$) is so that the stationary distribution of σ_i^2 is independent of λ_i .

Non-Gaussian OU processes

- Must have that $z_i(t)$ is a subordinator ie a positive, increasing, finite variation process.
- Note that the SDE => $\sigma_i^2(t)$ only jumps up and then between jumps, it is deterministic and decays exponentially with time.
- Note $\sigma_i^2(t)$ is always strictly positive since it is bounded below by

$\exp(-\lambda_i(t-t_0))\sigma_i^2(t_0)$ (which is > 0).

Calculus for non-Gaussian OU processes

- Barndorff-Nielsen and Shephard show that the following important relationship must hold:

  Levy density(x) of BDLP = -d/dx(x * Levy density(x) of law of stationary distribution)

- There is some freedom in what to choose as the BDLPs $z_i(t)$.
- Two main choices for $z_i(t)$.
- 1. Gamma-OU in which stationary distribution of $\sigma_i^2(t)$ is gamma distributed .
- Note that in the gamma-OU terminology, gamma refers to the stationary distribution – gamma does not refer to the distribution of the BDLP.
- In fact, the calculus result from last page =>

  BDLP $z_i(t)$ is compound Poisson with exponentially distributed jumps => $z_i(t)$ is finite activity Levy process.

- 2. TS-OU in which stationary distribution of $\sigma_i^2(t)$ is TS (Tempered Stable) distributed.
- The characteristic function of the Tempered Stable (TS) distribution $TS(\kappa, a, b)$, with $a > 0$, $b \geq 0$ and $0 < \kappa < 1$, is $\exp\left(ab - a\left(b^{1/\kappa} - 2iu\right)^{\kappa}\right)$.
- The distribution is infinitely divisible and we define a TS stochastic process as the process which starts at zero, has independent and stationary increments and for which over the time period from s to $t+s$, the increment in the TS process follows a $TS(\kappa, at, b)$ distribution.

TS process continued

- The levy density of the $TS(\kappa, a, b)$ process is:

$$a2^{\kappa}\frac{\kappa}{\Gamma(1-\kappa)}x^{-\kappa-1}\exp\left(-\frac{1}{2}b^{1/\kappa}x\right)1_{(x>0)}$$

- A TS process is related to the CGMY process in that a CGMY process can be thought of as the difference of two independent TS processes – or, to put it in a different way, it is the sum of two independent TS processes where one independent TS process provides positive (upward) jumps and the other has a minus sign in front of it so that it provides negative (downwards) jumps.
- => CGMY can jump both up and down whereas TS only jumps up.

TS-OU continued

- The result I quoted in the Calculus earlier implies that for the TS-OU specification:
- The BDLP $z_i(t)$ is the sum of a compound Poisson process and a TS $TS(\kappa, a\kappa, b)$ process.

  Since a TS process has infinite activity so does the BDLP $z_i(t)$.

  In fact, from the calculus for non-Gaussian OU processes, it is easy to show that the compound Poisson process component is a Poisson sum of gamma distributed random variables.

- We introduce M such BDLP (background driving Levy processes) processes $z_i(t)$ (all independent of each other).

- Define $\overline{\sigma}^2(t) \equiv \sum_{i=1}^{M} \varpi_i \sigma_i^2(t)$ where $\varpi_i \geq 0$

  for each i , $i = 1,\ldots,M$

  ie $\overline{\sigma}^2(t)$ is a weighted sum of stochastic variance terms, each of which follows a non-Gaussian OU process.

- Our stochastic volatility term will simply be:

$$\overline{\sigma}(t) = \sqrt{\sum_{i=1}^{M} \varpi_i \sigma_i^2(t)}$$

- Notation:
- We denote the initial time (today) by t_0. We denote the spot fx rate, at time t, by $X(t)$, (number of units of domestic currency per unit of foreign currency). We denote the (cont. compounded) short rates, at time t, in the domestic and foreign currencies by $r(t)$ and $r_f(t)$ respectively.

SDE for BNS 2 specification

- We assume for specification BNS2 that the SDE (under an EMM) for the (log) spot fx rate is:

$$d(\ln X(t)) = \left(r(t) - r_f(t) - \frac{1}{2}\sigma_X^2(t) - \frac{1}{2}\overline{\sigma}^2(t) - \sum_{i=1}^{M} \lambda_i \log E_0\left[\exp(\rho_i z(1))\right] \right) dt$$

$$+ \sigma_X(t) dW_X(t) + \overline{\sigma}(t) d\overline{W}(t) + \sum_{i=1}^{M} \rho_i dz_i(\lambda_i t)$$

Note $dW_X(t)$ and $d\overline{W}(t)$ are INDEPENDENT Brownian increments and $\sigma_X(t)$ is a purely deterministic function of t (in practice, either constant or piecewise constant) .

- We have in mind the case where $M=2$, with $\rho_1 \geq 0$, $\rho_2 \leq 0$, $\varpi_1 = \frac{1}{2}$, $\varpi_2 = \frac{1}{2}$

(this is the specification we will consider (BNS2) but clearly other choices are possible).

This specification can create a volatility smile which is roughly symmetric on the average (as fx smiles typically are) as well as both a positive and negative “leverage” effect.

The spot fx rate can clearly have both positive (upward) and negative (downward) jumps.

When there are jumps in the fx rate, there are (in general) simultaneous jumps in the volatility (this seems realistic and highly intuitive).

- The previous specification, which we referred to as BNS2, is intuitive and flexible and can broadly capture the same empirical features as Heston + jumps.
- However, it CANNOT capture stochastic skew (ie stochastic risk-reversals). To do that we need another specification which we refer to as TCLP2 (short-hand for 2 Levy processes time-changed by 2 other Levy processes).

TCLP2 specification

- Instead of time-changing a Brownian motion, independently time change two independent Levy processes – one of which can only jump up and the other can only jump down.
- We denote the instantaneous activity rate, at time t, by $v_i(t)$ (intuitively this describes the flow of business time as opposed to calendar time). We assume that $v_i(t)$ follows a non-Gaussian OU process of the same form as before:

- $dv_i(t) = -\lambda_i v_i dt + dz_i(\lambda_i t)$ with

  $v_i(t_0) > 0$ and $z_i(\lambda_i t_0) \equiv 0$.

  Actually to avoid a degeneracy in the calibration, we set (without any loss of generality): $v_i(t_0) = 1$

  We define $V_i(t) = \int_{t_0}^{t} v_i(s) ds$

- Can easily show from the SDE that for times t_1 and t_2 (where $t_2 > t_1$):

$$v_i(t_2) = v_i(t_1)\exp(-\lambda_i(t_2 - t_1)) + \exp(-\lambda_i t_2)\int_{t_1}^{t_2}\exp(\lambda_i s)dz_i(\lambda_i s)$$

and

$$V_i(t_2) - V_i(t_1) = \int_{t_1}^{t_2} v_i(s)ds = \frac{1}{\lambda_i}v(t_1)(1 - \exp(-\lambda_i(t_2 - t_1)))$$

$$+\frac{1}{\lambda_i}\int_{t_1}^{t_2}(1 - \exp(-\lambda_i(t_2 - s)))dz_i(\lambda_i s)$$

- Then, for specification TCLP2, we assume that the dynamics (under an EMM) of the spot fx rate are:

$$d(\ln X(t)) = \left(r(t) - r_f(t) - \frac{1}{2}\sigma_X^2(t) - \sum_{i=1}^{2}\lambda_i \log E_0[\exp(\rho_i z(1))] \right) dt$$

$$+\sigma_X(t) dW_X(t) + \sum_{i=1}^{2}\rho_i dz_i(\lambda_i t) + \sum_{i=1}^{2}\left(\eta_i dJ_i(V_i(t)) - dV_i(t) \log E_0\left[e^{\eta_i J_i(1)}\right]\right)$$

where $J_i(t)$ are spectrally positive Levy processes (can only jump up) and where $\eta_1 = +1$, $\eta_2 = -1$. As for ρ_1 and ρ_2, could work with either (1) $\rho_1 \equiv 0$, $\rho_2 \equiv 0$ or (2) $\rho_1 > 0$, $\rho_2 < 0$

Monte Carlo Simulation

- Now it is easy to simulate all these processes thanks to Rosinski (2001), (2002) We'll focus on simulating TCLP2 as this is harder.
- Define the state variables $Y_i(t) \equiv \int_{t_0}^{t} dz_i(\lambda_i s)$ and $X_i(t) \equiv \int_{t_0}^{t} \exp(\lambda_i s) dz_i(\lambda_i s)$ in terms of which the integrated stochastic activity rate between two times t_1 and t_2 can be written:

$$V_i(t_2) - V_i(t_1) = \int_{t_1}^{t_2} v_i(s) ds = \frac{1}{\lambda_i} v(t_1)(1 - \exp(-\lambda_i(t_2 - t_1)))$$

$$+ \frac{1}{\lambda_i}(Y_i(t_2) - Y_i(t_1) - \exp(-\lambda_i t_2)\{X_i(t_2) - X_i(t_1)\})$$

- Then simulate spot fx rate via:

$$\ln X(t_2) - \ln X(t_1) = \int_{t_1}^{t_2} \left(r(s) - r_f(s) - \frac{1}{2}\sigma_X^2(s) - \sum_{i=1}^{2} \lambda_i \log E_0\left[\exp(\rho_i z(1))\right] \right) ds$$

$$+ \int_{t_1}^{t_2} \sigma_X(s) dW_X(s) + \sum_{i=1}^{2} \int_{t_1}^{t_2} \rho_i dz_i(\lambda_i s) + \sum_{i=1}^{2} \left(\int_{t_1}^{t_2} \eta_i dJ_i(V_i(s)) - \int_{t_1}^{t_2} \log E_0\left[e^{\eta_i J_i(1)}\right] dV_i(s) \right)$$

$$= \int_{t_1}^{t_2} \left(r(s) - r_f(s) - \frac{1}{2}\sigma_X^2(s) - \sum_{i=1}^{2} \lambda_i \log E_0\left[\exp(\rho_i z(1))\right] \right) ds + \int_{t_1}^{t_2} \sigma_X(s) dW_X(s)$$

$$+ \sum_{i=1}^{2} \rho_i \left(Y_i(t_2) - Y_i(t_1) \right) + \sum_{i=1}^{2} \eta_i \int_{V_i(t_1)}^{V_i(t_2)} dJ_i(s) - \sum_{i=1}^{2} \left(\log E_0\left[e^{\eta_i J_i(1)}\right] \left(V_i(t_2) - V_i(t_1) \right) \right)$$

- So we need to simulate:

$$Y_i(t) \equiv \int_{t_0}^{t} dz_i(\lambda_i s) \quad , \quad X_i(t) \equiv \int_{t_0}^{t} \exp(\lambda_i s) dz_i(\lambda_i s)$$

Use results from Rosinski (2001) and Rosinski (2002).

To simulate a $TS(\kappa, a, b)$ process do:

$$\int_{t_1}^{t_2} f(\lambda s)\,dJ(\lambda s) = \sum_{j=1}^{\infty} 2\min\left\{\left(\frac{a\lambda(t_2 - t_1)}{b_j\Gamma(1-\kappa)}\right)^{1/\kappa}, \left(\frac{e_j v_j^{1/\kappa}}{b^{1/\kappa}}\right)\right\} f\left(\lambda(t_2 - t_1)u_j\right)$$

- where e_j are exp(1), u_j and v_j are uniform U(0,1), b_j are the arrival times of jumps of a Poisson process with unit intensity (ie successive partial sums of exponentials with unit mean),
- And where $f(\lambda s)$ is any deterministic function.

- Note each series of random numbers is independent of each other.

- Simulate a $TS(\kappa, a\kappa, b)$ process using the last formula (with $f(\lambda s) \equiv \exp(\lambda s)$ and $f(\lambda s) \equiv 1$).

 Add on a compound Poisson with gamma distributed jumps $\int_{t_1}^{t_2} f(\lambda s) dN(\lambda s) = \sum_{j=1}^{N} \Gamma\left(1-\kappa, \frac{1}{2} b^{1/\kappa}\right) f(\lambda(t_2 - t_1) u_j)$

- => get simulated $Y_i(t) \equiv \int_{t_0}^{t} dz_i(\lambda_i s)$, $X_i(t) \equiv \int_{t_0}^{t} \exp(\lambda_i s) dz_i(\lambda_i s)$

 and hence $V_i(t_2) - V_i(t_1)$ and hence $V_i(t_2)$, for each i .

- To simulate $\int_{V_i(t_1)}^{V_i(t_2)} dJ_i(s)$ use the infinite series again

$$\int_{t_1}^{t_2} f(\lambda s)\,dJ(\lambda s) = \sum_{j=1}^{\infty} 2\min\left\{\left(\frac{a\lambda(t_2-t_1)}{b_j\Gamma(1-\kappa)}\right)^{1/\kappa}, \left(\frac{e_j v_j^{1/\kappa}}{b^{1/\kappa}}\right)\right\} f(\lambda(t_2-t_1)u_j)$$

with t_1 and t_2 replaced by $V_i(t_1)$ and $V_i(t_2)$ respectively (and this time with $\lambda \equiv 1$ and $f(\lambda s) \equiv 1$).

Again:

$$\int_{t_1}^{t_2} f(\lambda s)\,dJ(\lambda s) = \sum_{j=1}^{\infty} 2\min\left\{\left(\frac{a\lambda t}{b_j\Gamma(1-\kappa)}\right)^{1/\kappa}, \left(\frac{e_j v_j^{\;1/\kappa}}{b^{1/\kappa}}\right)\right\} f\big(\lambda(t_2 - t_1)u_j\big)$$

- Infinite series of terms which are positive and eventually decreasing. Can truncate after finite number of terms once a desired convergence tolerance (eg 10^{-11} or 10^{-14}) has been achieved.
- One issue: How many terms are needed?

- Answer: It is very, very sensitive to the value of κ .

  I chose some values of κ and held $b^{1/\kappa}$ constant. Then over 1000 simulations computed the average number of terms in the infinite series needed to converge to three different convergence tolerances (10^{-08}, 10^{-11}, 10^{-14}) for a TS-OU process over a time period of one year.

  The results are:

	convergence tolerance		
	1.E-08	1.E-11	1.E-14
kappa	mean number of terms		
0.121	3	7	17
0.321	59	498	4930
0.521	548	24077	?
0.721	5334	481480	?
0.921	32521	?	?

Simulating Gamma-OU and TS-OU cases of BNS2

- TS-OU is very similar to TCLP2 but now we are time-changing a Brownian motion.
- Gamma-OU is even easier (just need to simulate compound Poisson).
- => can simulate all three different models / specifications (Gamma-OU BNS2, TS-OU BNS2, TCLP2), without discretization error, with just 20 to 25 lines of code (cf Heston).
- And that's it !

Barrier options

- Suppose we wish to price barrier options. For discretely monitored barriers, there are no additional issues.
- What about barriers which are monitored continuously?
- In the case of Gamma-OU BNS2, we are simulating finite activity processes.
- => easy way to price barrier options by Monte Carlo simulation.

Barrier options

- 1./ Simulate the fx rate to some chosen fixed dates.
- 2./ Simulate the actual times of the jumps.
- 3./ Between jumps, the volatility is deterministic (actually decays exponentially).
- 4./ Then we need probability that a Brownian bridge process constructed from a BM with a deterministic non-constant vol hits a (in practice, usually a linear) barrier:
- 5./ But this is the same as the probability that a Brownian bridge process constructed from a BM with a constant volatility hits a (definitely) NON-LINEAR barrier:
- Then we are done.
- Note the chosen fixed dates in part 1./ may correspond to dates at which the risk-free rates and/or the purely deterministic vol $\sigma_X(t)$ changes.

Brownian Bridge

- Actually, the probability that a Brownian bridge process constructed from a BM with a constant volatility hits a non-linear barrier is not, in general, known in closed form but there are accurate and very fast numerical approximations (Roberts and Shortland (1995) and Potzelberger and Wang (2001)).
- The accuracy of these approximations will deteriorate over long time-steps which is a reason to make them not too long.

Barrier options with TS-OU BNS2

- What about barrier options for the TS-OU version of BNS2? Wrote down infinite series:

$$\int_{t_1}^{t_2} f(\lambda s)dJ(\lambda s) = \sum_{j=1}^{\infty} 2\min\left\{\left(\frac{a\lambda t}{b_j\Gamma(1-\kappa)}\right)^{1/\kappa}, \left(\frac{e_j v_j^{1/\kappa}}{b^{1/\kappa}}\right)\right\} f(\lambda(t_2-t_1)u_j)$$

- Note $(t_2 - t_1)u_j$ are the simulated times of the jumps. Although there are actually an infinite number of jumps in a finite period of time, our truncation (at some pre-determined but tiny tolerance) makes the simulated number of jumps finite.
- Then proceed as with Gamma-OU version.
- But for kappa close to one, would be very slow.

- For smallish values of kappa (< 0.3), have priced barrier options with convergence tolerances of

  either (1) 10^{-11} or (2) 10^{-14}

  and with max fixed time steps of

  either (a) one week or (b) three months.

  With, eg, 650 million runs, one year option, there is no discernable difference in price estimates with the two different convergence tolerances and with the two different maximum lengths of fixed time-steps, suggesting this algorithm is very accurate.
- Where possible, it is desirable to calibrate model not only to vanillas but also to actively-traded barrier options (eg double no-touch). This is certainly possible if we assume only smallish values of kappa (eg < 0.3) are allowed.

Calibration to market data

- Practical success of the algorithm depends on kappa being less than about 0.3, say.
- Calibrated Gamma-OU BNS2, TS-OU BNS2, TCLP2 and the Carr-Wu model to

  cable (USD/STG), USD/EUR, SFR/USD options (5 different strikes, 12 different maturities) as of 22 January 2007 using Fourier inversion.
- Note to reduce the number of parameters to be determined in the calibration, I pre-set kappa to be the same for each one of the pair of TS-OU processes and each one of the pair of TS processes which are time-changed (for TCLP2).

Calibrated kappa values

	TS-OU BNS2	TCLP2	Carr-Wu (alpha/Y)
Cable			
Kappa of OU processes	0.121	0.227	N/A
Kappa of indep. processes $J_i(t)$	N/A	0.191	0.248
USD/EUR			
Kappa of OU processes	0.123	0.121	N/A
Kappa of indep. processes $J_i(t)$	N/A	0.214	0.298
SFR/USD			
Kappa of OU processes	0.162	0.151	N/A
Kappa of indep. processes $J_i(t)$	N/A	0.198	0.262

Calibration to market data

- Note also set lambda parameter to be the same across the two different processes.
- For Gamma-OU, had 10 parameters to estimate ie

  $\rho_1, \rho_2, a_1, a_2, b_1, b_2, \sigma_1(t_0), \sigma_2(t_0), \sigma_X, \lambda_1 \equiv \lambda_2$
- For TS-OU, had 11 parameters to estimate ie

  $\rho_1, \rho_2, a_1, a_2, b_1, b_2, \sigma_1(t_0), \sigma_2(t_0), \sigma_X, \lambda_1 \equiv \lambda_2, \kappa_1 \equiv \kappa_2$
- For TCLP2, had 12 parameters to estimate ie

  σ_X , for the OU processes $a_1, a_2, b_1, b_2, \lambda_1 \equiv \lambda_2, \kappa_1 \equiv \kappa_2$

for the independent processes $a_1, a_2, b_1, b_2, \kappa_1 \equiv \kappa_2$

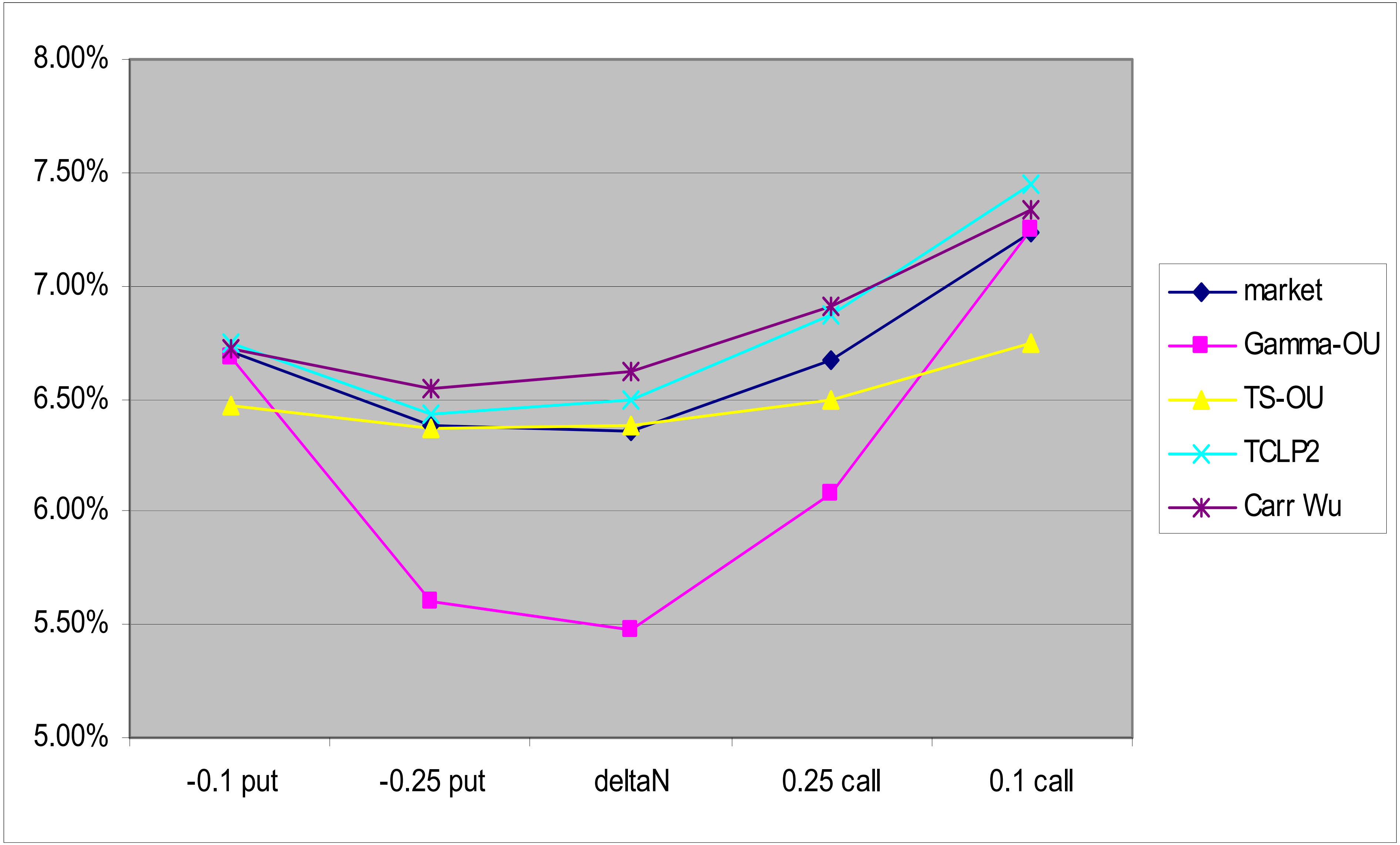
Note we set $\rho_1 \equiv 1, \rho_2 \equiv 1,$ (slightly arbitrary).

- For Carr and Wu, had 10 parameters to estimate.

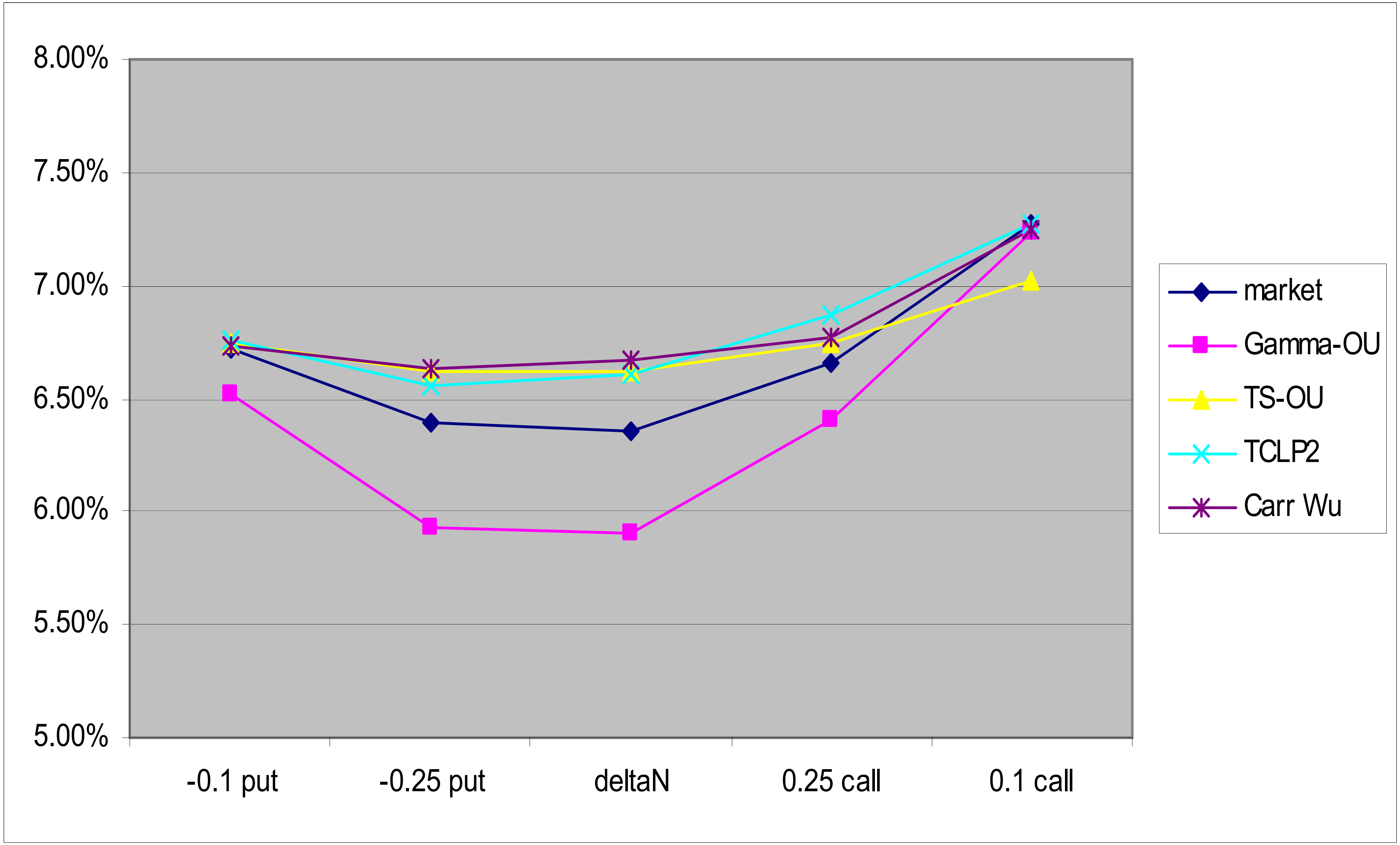
Calibration continued

- Note that it would be possible to do a second "refinement" fit by utilising the Brownian motion with deterministic volatility (ie the $\sigma_X(t)dW_X(t)$ term). We could make $\sigma_X(t)$ piecewise constant in order to, for example, give a perfect fit to at-the-money options although we don't do this here (where we did assume $\sigma_X(t) \equiv \sigma_X$, a constant, $0.03 < \sigma_X < 0.0575$).
- Results are:

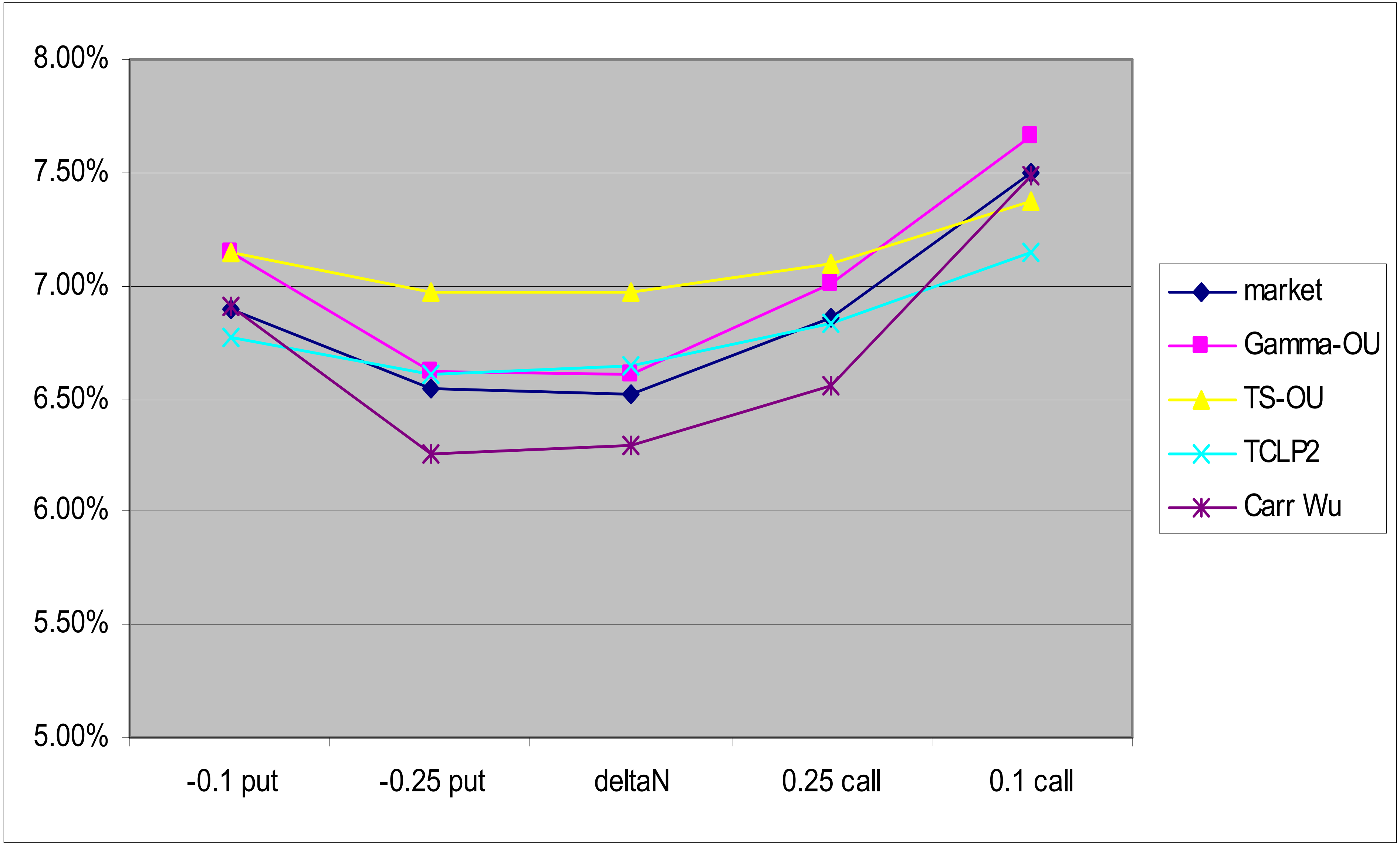
3 month cable USD/STG options



6 month cable USD/STG options



1 year cable USD/STG options



Summary of fits

	Gamma-OU BNS2	TS-OU BNS2	TCLP2	Carr-Wu
Cable	3.27	2.67	1	1.37
USD/EUR	5.52	1.05	1	1.04
SFR/USD	6.02	1	1.13	1.01

(Note fits are scaled to one so 1 indicates the best fit out of the four different models for each currency pair and the bigger the number the worse the fit)

Cross-currency/ interest-rate hybrid derivatives/long-dated fx options

- Can assume interest-rates in both domestic and foreign currencies are stochastic and driven by a Gaussian HJM model.
- Then everything goes through pretty much as before provided we assume that interest-rates are independent of the Brownian motion with the stochastic vol. term.
- We can capture correlation between rates and fx through the Brownian motion term with deterministic volatility (ie $\sigma_X(t)dW_X(t)$ term).

Cross-currency/interest-rate hybrid derivatives/ long-dated fx options

- We could add jumps and stochastic volatility (as for fx, via a non-Gaussian OU process driven by a spectrally positive Levy process) to the dynamics of interest-rates (to capture IR vol. skew/smile).
- As a topic for further research, we mention that we could make some of the jumps common (through the weightings ϖ_i) to both interest-rates and fx in order to capture co-dependence of movements between rates and fx (ie use a common or partially common stochastic time-change).

Conclusions

- Stochastic time-changes using non-Gaussian OU processes leads to a wide class of models which are:
- Intuitive, easy to implement.
- Allows for stochastic volatility and jumps (with either finite or infinite activity)
- Can give good fits to market prices of options.
- A subset of the models include ability to capture stochastic skew.
- Monte Carlo simulation, without discretisation error, is very straightforward => straightforward exotics pricing.
- Works best if kappa parameter is not too large but this seems to the case in practice.

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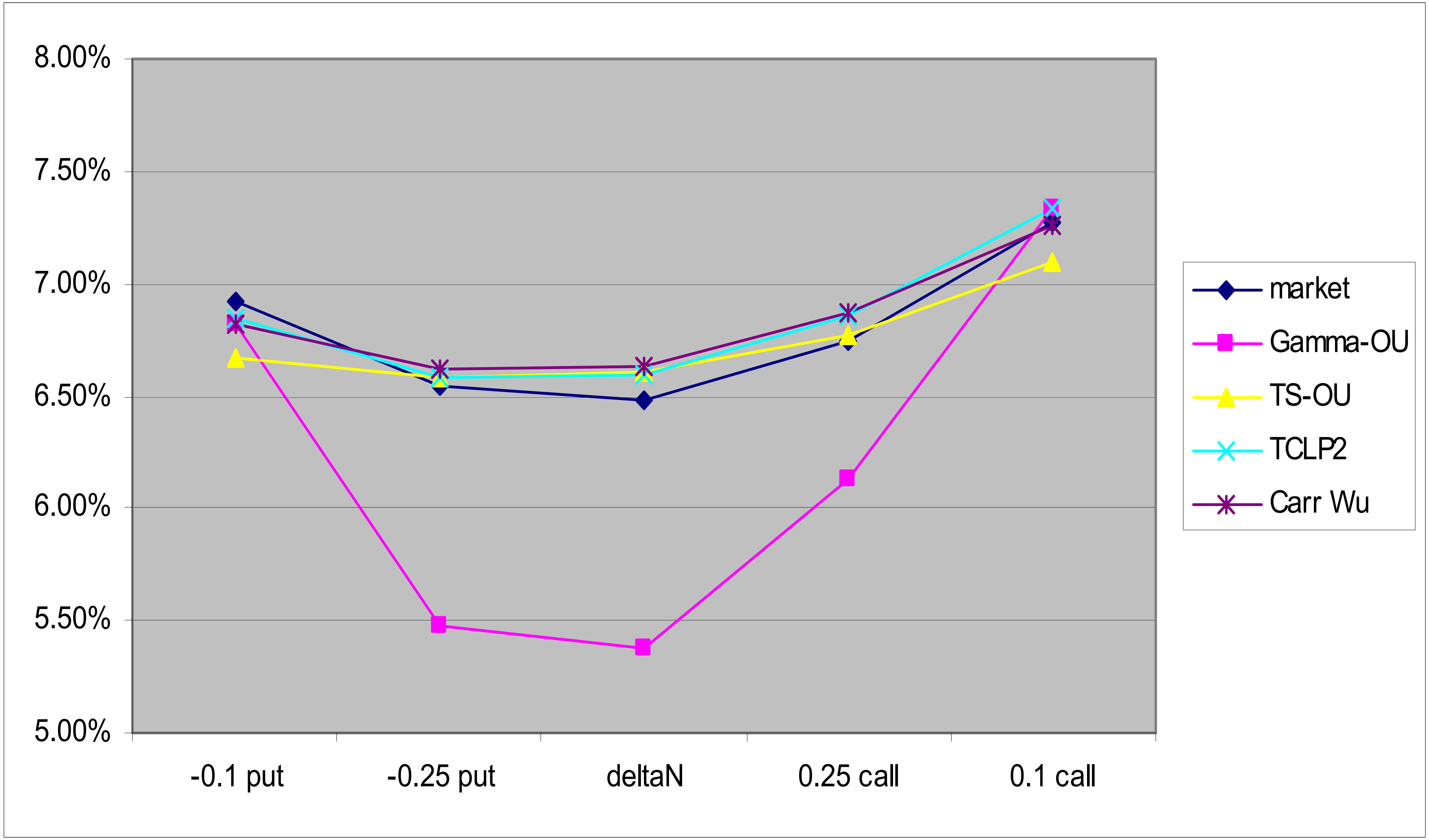
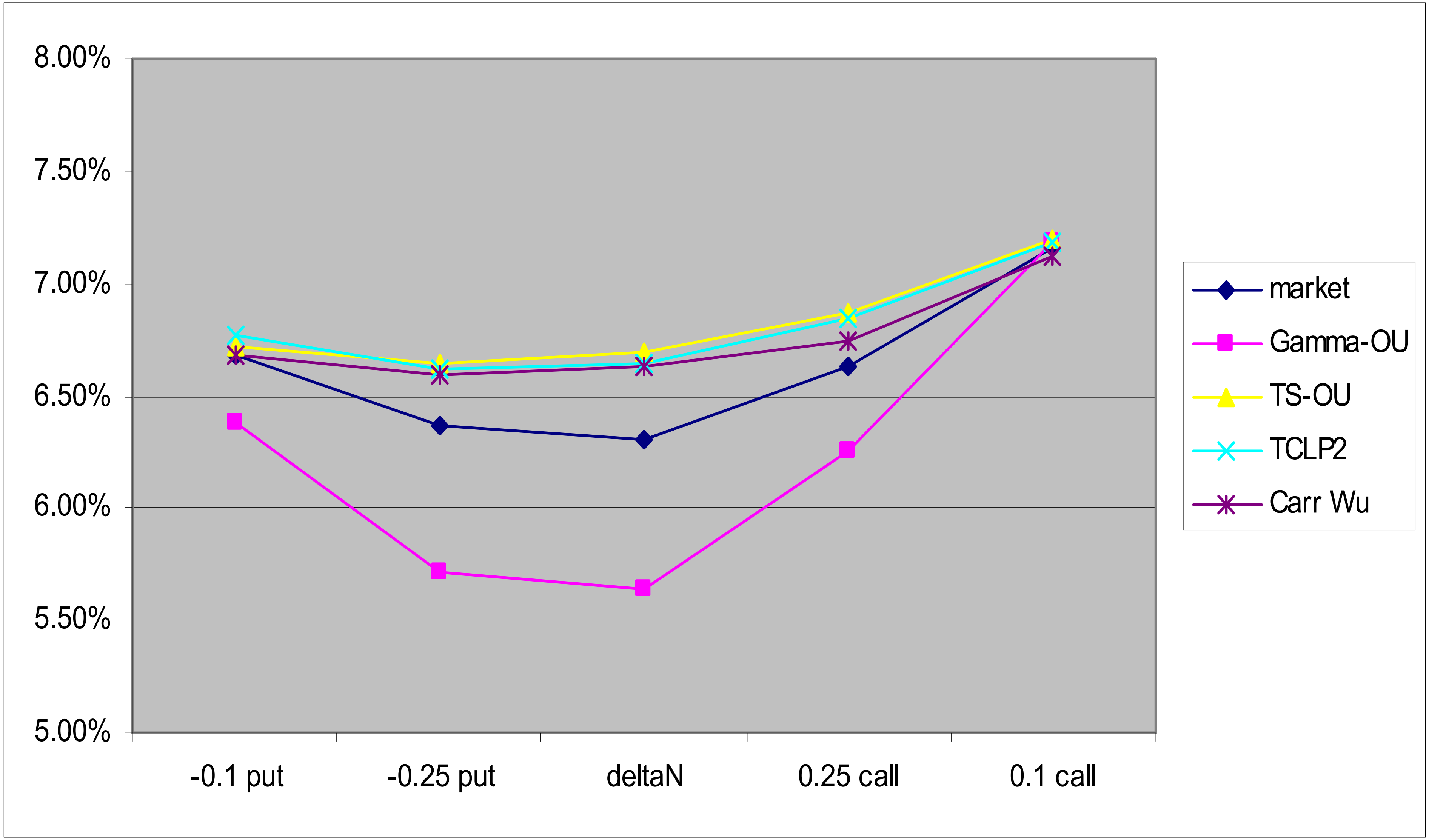
3 month USD/EUR options



6 month USD/EUR options



1 year USD/EUR options