Commodity and Commodity Hybrid Derivatives

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Risk South Africa, 15th March 2007

Acknowledgements

- We thank Simon Babbs, Michael Dempster and Farshid Jamshidian for comments which have significantly enhanced our work. We also thank Peter Carr, Darrell Duffie, Stewart Hodges, Andrew Johnson, Daryl Mcarthur, Anthony Neuberger, Olga Pavlova, Chris Rogers and Nick Webber.

Modelling commodity prices with a multi-factor jump-diffusion model

This presentation partially draws on my papers "**A multi-factor jump-diffusion model for Commodities**" (accepted for publication), "**Pricing a class of exotic commodity options in a multi-factor jump-diffusion model**" (submitted for publication) and "**Valuing Inflation Futures Contracts**" (accepted for publication – coming out in Risk in March 2007) as well as "**Commodity options optimized**" (Risk Magazine, May 2006 p72-77).

Empirical observations on the commodities markets 1

- Spot commodity prices exhibit mean reversion.
- Convenience yields are usually highly volatile.
- Futures (and forward) commodity prices have instantaneous volatilities which usually (but not always) decline with increasing tenor.
- Implied volatilities in options markets often exhibit sharp skews (usually skews but sometimes smiles).
- Jumps are somewhat more common and certainly much larger in magnitude than in other markets (eg equities or fx).

Empirical observations on the commodities markets 2

- Consider Gold. When there are jumps in the market price of Gold, the jumps cause a parallel shift in the whole forward (or futures) curve.
- Rather like forward fx rates.
- “Gold trades somewhat like a currency”.
- BUT it’s a completely different story for other commodities….

Empirical observations on the commodities markets 3

….. a defining feature in nearly all other commodities markets (especially energy-related commodities such as Crude Oil, Natural Gas and Electricity) is that when there is a jump, the spot and short-dated futures (or forward) prices jump by a large amount but (very) long-dated contracts hardly jump at all (to our knowledge no existing models have accounted for this feature).

Evidence for this (part one) from Crude Oil Futures

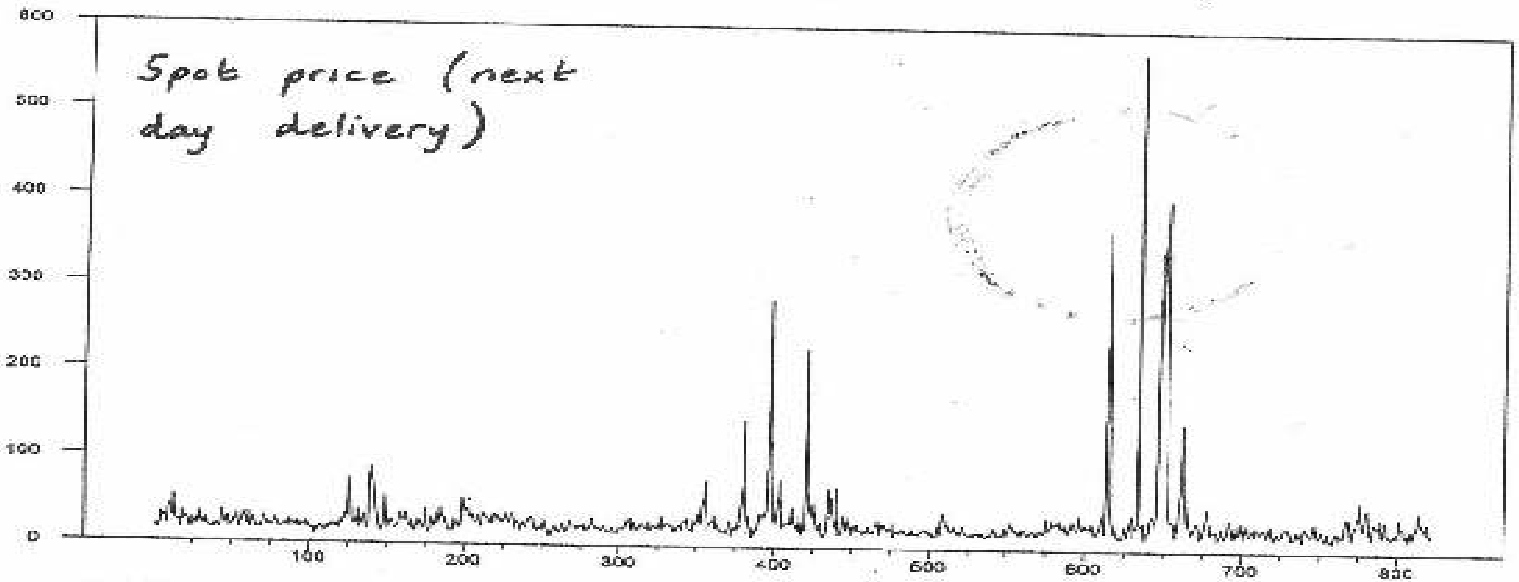
Iraq’s invasion of Kuwait in August 1990

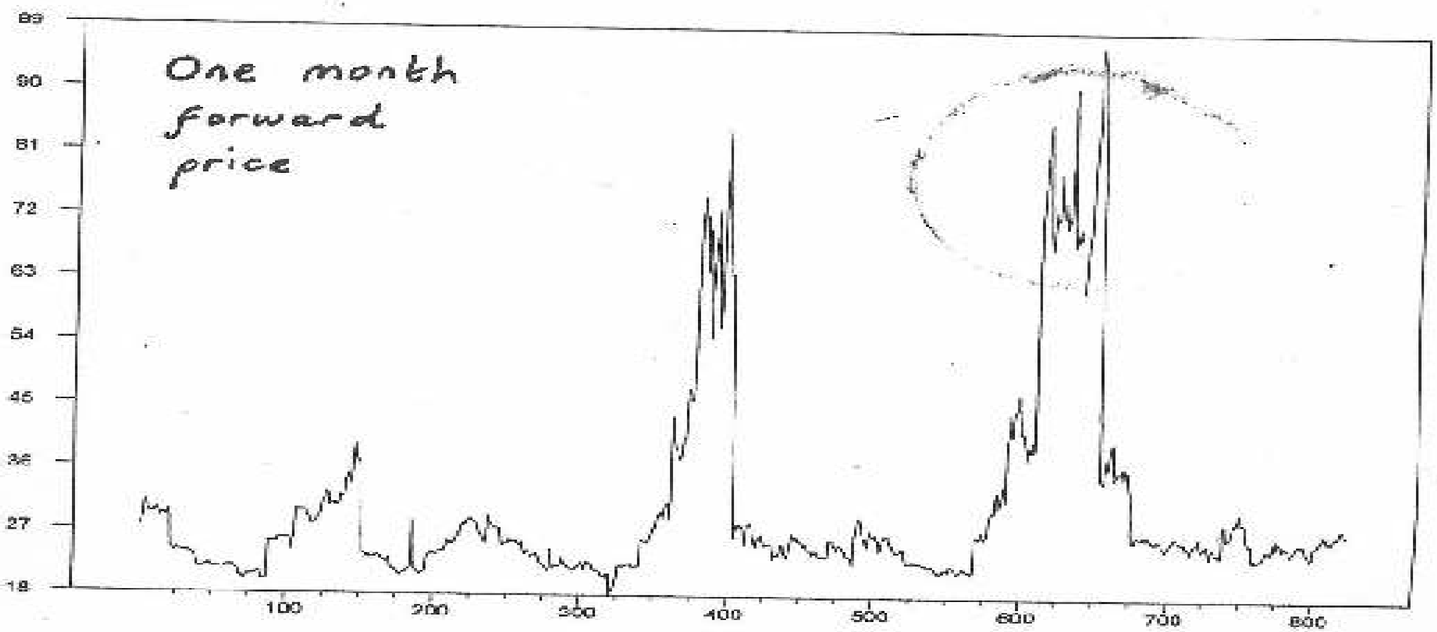
- Immediately before Iraq’s invasion of Kuwait in August 1990, the price of crude oil futures for next month delivery was approximately 17 dollars per barrel whilst the price of crude oil futures for 18 month delivery was approximately 19 dollars per barrel.
- Over the next 5 1/2 months (up to the start of the Gulf War), the (rolling) price for next month delivery, on two separate occasions, touched 40 dollars per barrel. In contrast, the (rolling) price of crude oil futures for 18 months delivery never went above 27 dollars per barrel during the whole period.

Evidence for this (part two) from electricity prices

- At the beginning of the second half of 1998, the prevailing price for electricity in the Pennsylvania-New Jersey-Maryland area of the U.S. was around 25 dollars per MWh for both spot (next day delivery) and one month forward delivery.
- During the course of the following six months, there were large jumps in the price of electricity which caused the spot price to rise above 350 dollars per MWh on three separate occasions. On each occasion that there was a jump in the spot price, a jump was also observed in the forward price of electricity. However, during this entire period, the forward price of electricity for one month delivery never exceeded 98 dollars per MWh.
- Note also that, on each of the three occasions the spot price jumped above 350 dollars per MWh, the spot price quickly (within two or three weeks) reverted back to a level below 40 dollars per MWh.

Price series PJM market.





Evidence for this (part three) from the market prices of options on crude oil futures.

- Later in this talk, I will present this evidence (after we look at the model in greater depth).
- Basically, we calibrate our model to the market prices of options and show that allowing for the feature, that short-dated futures contracts can jump by more than long-dated futures contracts, gives a better fit.

Evidence (part four)

- This is more of a mind experiment.
- Suppose now we have our calibrated model parameters and we wish to price a European option, whose payoff is the greater of zero or the ratio of the price of a futures contract with a further (at option maturity) one month to delivery divided by the price of a futures contract with a further (at option maturity) two years to delivery minus a fixed strike. This is a simple type of exotic option on the slope of the term structure of futures commodity prices. If we assume (as the existing literature does) that, when there are jumps, futures contracts of all maturities jump by the same proportional amount then, it is easy to show that, the price of this exotic option, given the model parameters, is indifferent to jumps. However, if we assume that the prices of short-dated futures contracts jump by more than long-dated contracts, then the price of the option will be influenced by jumps, which is what one would expect to be the case given the empirically observed behaviour described above.

Commodities

- Now let’s start to look at our model.
- We would like to capture the stylised empirical features of the commodities which we have just noted.
- We want a no-arbitrage model which automatically fits the initial term structure of futures (or forward) commodity prices.

Key Assumptions

- We make the standard assumptions ie the market is frictionless, (ie no bid-offer spreads, continuous trading is possible, etc) and arbitrage-free.
- No arbitrage => existence of an equivalent martingale measure (EMM).
- In this talk, we work exclusively under the (or a) EMM.

Stochastic Interest-rates

- We denote the (continuously compounded) risk-free short rate, at time t , by $r(t)$ and we denote the price of a zero coupon bond, at time t maturing at time T by $P(t,T)$. We assume that bond prices follow the extended Vasicek (Hull-White) process, namely,

$$\frac{dP(t,T)}{P(t,T)} = r(t)\,dt + \sigma_P(t,T)\,dz_P(t)$$

$$\sigma_P(t,T) \equiv \frac{\sigma_r}{\alpha_r}\left(1-\exp\left(-\alpha_r(T-t)\right)\right) \quad \text{where}$$

$\sigma_r > 0 \qquad \alpha_r > 0 \qquad$ are constants.

The model

- Let us denote the futures commodity price at time t for delivery at time T by $H(t,T)$

- We take as given our initial (ie at time t_0) term structure of futures commodity prices.
- Futures prices are martingales under the EMM. (Cox et al. (1981)).

The model

- Then we assume that the dynamics of futures commodity prices in the EMM are:

$$\frac{dH(t,T)}{H(t,T)}=\sum_{k=1}^{K}\sigma_{Hk}(t,T)\,dz_{Hk}(t)-\sigma_P(t,T)\,dz_P(t)$$

$$+\sum_{m=1}^{M}\left(\exp\left(\gamma_{mt}\exp\left(-\int_t^T b_m(u)du\right)\right)-1\right)dN_{mt}$$

$$-\sum_{m=1}^{M}\lambda_m(t)E_{Nmt}\left(\exp\left(\gamma_{mt}\exp\left(-\int_t^T b_m(u)du\right)\right)-1\right)dt$$

- K is the number of Brownian factors (for example, 1, 2, 3 or 4).
- The form of the volatility functions $\sigma_{Hk}(t,T)$ can be somewhat general at this time but we assume they are deterministic.
- The Brownian motions can all be correlated.
- M is the number of Poisson processes.

Jump processes

- For each m, $m = 1, \ldots, M$, $\lambda_m(t)$ are the (assumed) deterministic intensity rates of the M Poisson processes.
- $b_m(u)$ for each m are non-negative deterministic functions. We call these the jump decay coefficient functions.
- γ_{mt} are the spot jump amplitudes.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.1 in the paper:
- The spot jump amplitudes are (known) constants. In this case, the jump decay coefficient functions $b_m(u)$ can be non-negative (but otherwise arbitrary) deterministic functions.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.2 in the paper:
- The spot jump amplitudes are assumed to be independent and identically distributed random variables (assumed independent of everything else). In this case, the jump decay coefficient functions must be equal to zero. ie $b_m(t) \equiv 0$ for all t

Multiple Poisson processes

- All satisfy either Assumption 2.1 or 2.2
- But, if we have more than one Poisson process, we can mix the assumptions
- Eg if there were 4 Poisson processes, we could have eg 3 satisfying assumption 2.1 and 1 satisfying Assumption 2.2

- Then by Ito's Lemma:

$$d\left(\ln H\left(t,T\right)\right) = \text{ (deterministic terms) dt } +$$

$$+\sum_{k=1}^{K}\sigma_{Hk}\left(t,T\right)dz_{Hk}\left(t\right)-\sigma_{P}\left(t,T\right)dz_{P}\left(t\right)$$

$$+\sum_{m=1}^{M}\gamma_{mt}\exp\left(-\int_{t}^{T}b_{m}\left(u\right)du\right)dN_{mt}$$

Implications

- Gas, Electricity: Short end of futures curve jumps a lot, long end hardly jumps at all (existing models do not seem to have this).
- Gold: Jumps are less of a feature (but they do happen).
- “Gold trades somewhat like a currency”.
- ie jumps cause parallel shift in futures (and forward) curve.

Model captures empirical features of the commodity and commodity options markets

- We show in the papers that our model can capture all the empirical features of the commodity and commodity options markets, that we mentioned a few slides ago, such as:
- Mean reversion
- Jumps
- Stochastic convenience yields

Can use Monte Carlo simulation to price nearly any type of derivative

- We show in the papers that it is relatively straightforward to use Monte Carlo simulation to price exotic commodity derivatives.
- Ditto, interest-rates, fx and inflation
- Hence ditto, hybrid derivatives involving commodities and any of the above.

What if there are two commodity prices?

- Consider two (arbitrary) commodities, labelled 1 and 2 on which there are correspondingly two futures contracts. We denote the futures price of Commodity i $i = 1,2$ at time t to time $T_{2,i}$ (ie the futures contract, into which Commodity i is deliverable, matures at time $T_{2,i}$) by $H_i(t, T_{2,i})$.

A class of exotic commodity options

- In the papers, we price a European-style option whose payoff is the greater of zero and a particular function involving the futures prices at times $T_{1,1}$ and $T_{1,2}$ of the futures contracts on Commodity 1 and Commodity 2 respectively.
- Choose $T_{1,2} \leq T_{1,1}$ (arbitrarily)
- The payoff is known at time $T_{1,1}$ but is paid at (a possibly later) time T_{pay}. Note $T_{pay} \geq T_{1,1} \geq T_{1,2}$.

A class of exotic commodity options

- More mathematically, we price a European-style option whose payoff is:

$$\max\left(\eta\left(\frac{H_1\left(T_{1,1},T_{2,1}\right)-K^*\left[H_2\left(T_{1,2},T_{2,2}\right)\right]^{\varepsilon}}{\left[H_2\left(T_{1,2},T_{2,2}\right)\right]^{\alpha}}\right),0\right)$$

at time T_{pay}

where K^* is a constant which might, for example, account for different units of measurement.

A class of exotic commodity options

- Also $\eta = 1$ if the option is a call and $\eta = -1$ if the option is a put.

Note ε and α are constants (need not be integers but, in practice, often might be).

Payoff again

- Payoff at time T_{pay}

$$\max\left(\eta\left(\frac{H_1\left(T_{1,1},T_{2,1}\right)-K^*\left[H_2\left(T_{1,2},T_{2,2}\right)\right]^{\varepsilon}}{\left[H_2\left(T_{1,2},T_{2,2}\right)\right]^{\alpha}}\right),0\right)$$

- Need $T_{pay} \geq T_{1,1} \geq T_{1,2}$, $T_{2,1} \geq T_{1,1}$, $T_{2,2} \geq T_{1,2}$
- Why consider this (slightly obscure) form?

Spread options

- General form: $\max\left(\eta\left(\frac{H_1(T_{1,1},T_{2,1})-K^*[H_2(T_{1,2},T_{2,2})]^{\varepsilon}}{[H_2(T_{1,2},T_{2,2})]^{\alpha}}\right),0\right)$

- Spread options on two different commodity futures: $\varepsilon=1$, $\alpha=0$
- Ratio spread or relative performance options on two different commodity futures: $\varepsilon=1$, $\alpha=1$
- For spread options on the spot, set

$$T_{1,1}\equiv T_{2,1}\equiv T_{1,2}\equiv T_{2,2}$$

Options on (slope of) futures curve

- General form: $\max\left(\eta\left(\dfrac{H_1(T_{1,1},T_{2,1})-K^*[H_2(T_{1,2},T_{2,2})]^{\varepsilon}}{[H_2(T_{1,2},T_{2,2})]^{\alpha}}\right),0\right)$
- Could have $H_1(\bullet,\bullet)\equiv H_2(\bullet,\bullet)$ ie actual same underlying commodity.
- Spread options on futures commodity curve:

  $\varepsilon=1$, $\alpha=0$
- Ratio spread or relative performance options on futures commodity curve: $\varepsilon=1$, $\alpha=1$

Forward-start and cliquet options on futures

- General form: $\max\left(\eta\left(\frac{H_1(T_{1,1},T_{2,1})-K^*\left[H_2(T_{1,2},T_{2,2})\right]^{\varepsilon}}{\left[H_2(T_{1,2},T_{2,2})\right]^{\alpha}}\right),0\right)$
- Could have $H_1(\bullet,\bullet)\equiv H_2(\bullet,\bullet)$ ie actual same underlying commodity.
- Forward start options on futures prices:

  $T_{1,2}$ strictly $< T_{1,1}$, $\varepsilon=1$ $\alpha=0$
- Ratio forward start (ie single-leg cliquets) on futures prices:
- $T_{1,2}$ strictly $< T_{1,1}$, $\varepsilon=1$ $\alpha=1$

Forward-start and cliquet options on spot

- General form: $\max\left(\eta\left(\dfrac{H_1(T_{1,1},T_{2,1})-K^*\left[H_2(T_{1,2},T_{2,2})\right]^{\varepsilon}}{\left[H_2(T_{1,2},T_{2,2})\right]^{\alpha}}\right),0\right)$
- Again $H_1(\bullet,\bullet)\equiv H_2(\bullet,\bullet)$ ie actual same underlying commodity. Put $T_{1,2}\equiv T_{2,2}$ and $T_{1,1}\equiv T_{2,1}$
- Forward start options on spot: Again

  $T_{1,2}$ strictly $< T_{1,1}$, $\varepsilon=1$ $\alpha=0$
- Ratio forward start (ie single-leg cliquets) on spot: Again
- $T_{1,2}$ strictly $< T_{1,1}$, $\varepsilon=1$ $\alpha=1$

Use of Fourier Transform methodology

- In the papers, using Fourier transform methods, we price options with the general payoff we have just discussed.
- Hence, we can also price the special cases ie spread options, options on slope of forward or futures curve, forward-start, cliquet options, etc.
- The algorithm is very fast eg can price an option in an average of 25 milliseconds.

Commodity – Inflation hybrid derivatives.

- A CPI index level can be seen as the local currency value (eg value in dollars) of a basket of "commodities" where some of the commodities may be services or finished goods (ie not necessarily primary commodities such as gold or crude oil).
- Rising commodity prices (esp. crude oil) in the 1970's caused higher inflation in the industrialised world.
- Rising prices of crude oil and other primary commodities in 2003 – 2006 now seem to be feeding through to inflation figures in U.K. and U.S.

Inflation

- It is natural to consider hybrid derivatives written jointly on the price of a commodity and an inflation index.
- We construct a model for inflation by using the "cross-currency model" analogy.
- We denote the spot inflation (CPI) index level, at time t, by $I(t)$.

Option on real return on a commodity (ratio)

- Consider a European option whose payoff at time T is:

$$\max\left(\eta\left(\frac{H(T,T)}{I(T)}-K^*\right),0\right)$$

- $\eta = +1/-1$ call/put

  K^* is a constant strike term (eg, might be the ratio of the (known) futures commodity price at the time the option was written divided by the (known) spot CPI index level at the time the option was written).

Option on real return on a commodity (difference)

- Consider a European option whose payoff at time T is:

$$\max\left(\eta\left(H(T,T)-K^{*}I(T)\right),0\right)$$

- $\eta=+1/-1$ call/put

  K^{*} is a constant multiplier (again, it might be the ratio of the (known) futures commodity price at the time the option was written divided by the (known) spot CPI index level at the time the option was written).

Option on real return on a commodity

- Can also price both of these types of options within our commodity model using a similar Fourier Transform based method as before (again it is very fast).
- Both of these types of option can be viewed as an option on the real (ie after adjustment for inflation) return on (or price of) a commodity (as opposed to options on the nominal return on (or price of) a commodity).
- Can also replace the single commodity by a commodity index (eg GSCI)

Standard (vanilla) options

- In the papers, we show that we can also price:
- Standard European options on futures.
- Futures-style options (both European and American) on futures contracts. (Note that many exchange traded options are of this type).
- Standard European options on forwards.
- Standard European options on the spot.

With Fourier Transform methodology:

- We can price 30 standard (vanilla) options in a total of less than 0.016 seconds
- => less than 1 millisecond per option.
- => we can determine the model parameters by calibrating the model to the market prices of standard options.

Calibration of our model to the market prices of options on crude oil futures.

- We calibrate two specifications of our model to the market prices of options on crude oil futures as of 25^{th} January 2005.
- The options were at 7 different strikes and 11 different maturities => 77 options in total.

Calibration cont'd

- First specification: We have the feature that short-dated futures contracts jump by less than long-dated futures contracts (which our model can allow for).
- Second specification: We assume all futures contracts jump by the same proportional amount (as existing models do).
- We claimed at the start of our talk that the first specification is better so lets have a look at the data.

Calibration cont'd

- Actually both specifications can give a reasonable fit but the fit for the first specification is much better:
- Maximum difference (which is a measure of the fitting error) between market and model implied volatilities across all 77 options is:

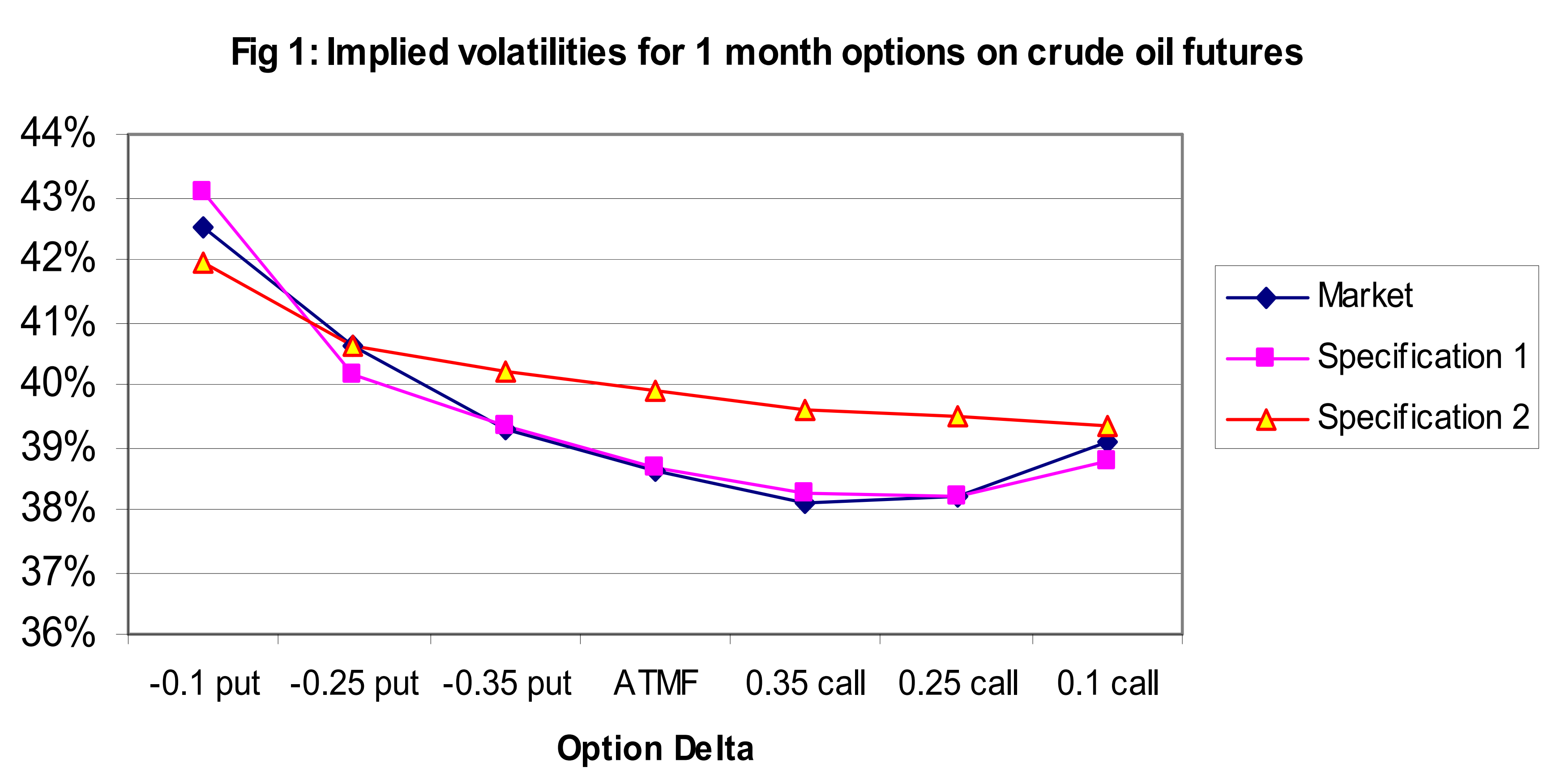
For First specification: 1.268 percentage points.

For Second specification: 1.980 percentage points.

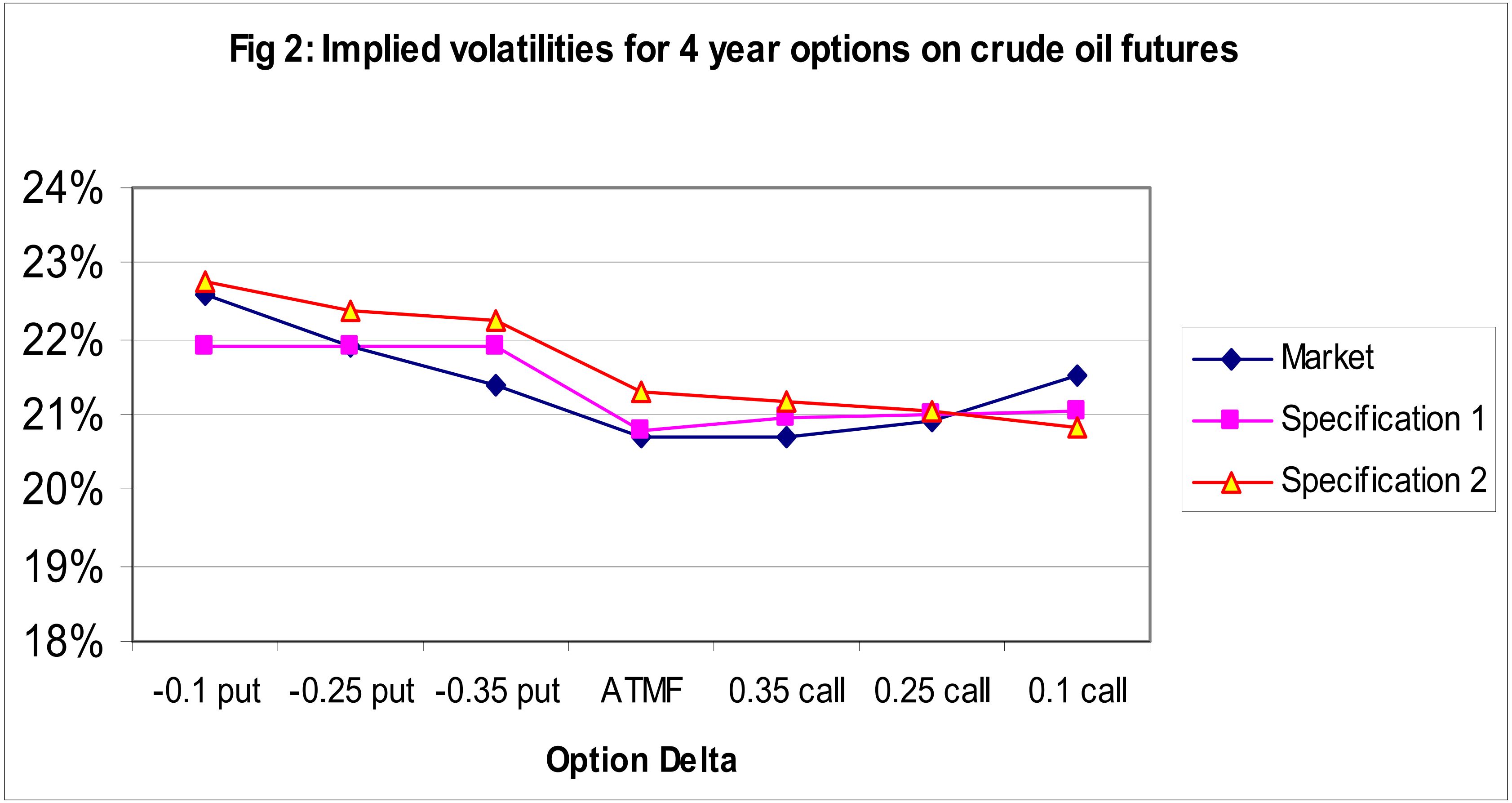
Calibration cont'd

- First specification gives a better fit (and also gives a better fit by other metrics such as mean squared proportional pricing error)
- ie we get a better fit when we allow short-dated futures contracts to jump by less than long-dated futures contracts.

One month options



Four year options



Summary

- The model is arbitrage-free.
- Automatically fits initial futures (or forward) commodity price curve.
- Captures empirical observations made about commodity prices (eg mean reversion, convenience yields (see paper)).
- Long-dated futures prices can jump by less than short-dated futures prices.
- Can price complex (exotic) commodity derivatives via Monte Carlo simulation.
- Can price some common exotic options using a Fourier Transform based algorithm.

Copies of the papers

- The papers which I mentioned earlier can be found on the website of the Centre for Financial Research at Cambridge University:

  http://mahd-pc.jbs.cam.ac.uk/seminar/2005-6.html

  or for "**Commodity options optimised**", Risk magazine, May 2006, p72-77

  or for "**Valuing Inflation Futures Contracts**" (to appear in Risk magazine in March 2007)