A multi-factor jump-diffusion model for Commodities

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Introducing a multi-factor jump-diffusion model for commodities

This presentation draws on my papers "A multi-factor jump-diffusion model for Commodities" and "Pricing commodity options in a multi-factor jump-diffusion model using Fourier Transforms" (submitted for publication).

Empirical observations on the commodities markets 1

- Spot commodity prices exhibit mean reversion.
- Futures (and forward) commodity prices have instantaneous volatilities which usually (not always) decline with increasing tenor.
- Jumps are somewhat more common and certainly much larger in magnitude than in other markets (eg equities or fx).

Empirical observations on the commodities markets 2

- A common feature in commodities (esp. Gas and Electricity) is that when there is a jump, the spot and short-dated futures (or forward) prices jump by a large amount but long-dated contracts hardly jump at all (to our knowledge no existing models have accounted for this feature).
- Convenience yields are usually highly volatile.

Commodities

- Now let's start to look at our model.
- We would like to capture the stylised empirical features of the commodities which we have just noted.
- We want a no-arbitrage model which automatically fits the initial term structure of futures (or forward) commodity prices.

Key Assumptions

- We assume the market is frictionless, (ie no bid-offer spreads, continuous trading is possible, etc) and arbitrage-free.
- No arbitrage => existence of an equivalent martingale measure (EMM).
- In this talk, we work exclusively under the (or a) EMM.

Spot commodity prices

- We denote the value of the commodity at time t by C_t. We define today to be time t_0 and the value of the commodity today is C_{t_0}.

- (Value of the commodity means "spot price" but in some markets spot is hard to define).

Spot commodity prices

- We do NOT assume that the spot is tradeable (except as the deliverable on a futures contract at maturity).

Stochastic Interest-rates

- We denote the (continuously compounded) risk-free short rate, at time t , by $r(t)$ and we denote the price of a zero coupon bond, at time t maturing at time T by $P(t,T)$. We assume that bond prices follow the extended Vasicek process, namely,

$$\frac{dP(t,T)}{P(t,T)} = r(t)\,dt + \sigma_P(t,T)\,dz_P(t)$$

$$\sigma_P(t,T) \equiv \frac{\sigma_r}{\alpha_r}\left(1 - \exp\left(-\alpha_r(T-t)\right)\right)$$

- where σ_r and α_r are positive constants.

- Define the state variable:

$$X_P(t) = \int_{t_0}^{t} \sigma_r \exp\left(-\alpha_r (t-s)\right) dz_P(s)$$

- ETS can write $r(t)$ and $P(t,T)$ in terms of

$X_P(t)$

The model

- Let us denote the futures commodity price at time t for delivery at time T by $H(t,T)$

- We take as given our initial (ie at time t_0) term structure of futures commodity prices.
- In the absence of arbitrage, $C_t = H(t,t)$
- Futures prices are martingales under the EMM. (Cox et al. (1981)).

The model

- Then we assume that the dynamics of futures commodity prices in the EMM are:

$$\frac{dH(t,T)}{H(t,T)} = \sum_{k=1}^{K} \sigma_{Hk}(t,T)\, dz_{Hk}(t) - \sigma_P(t,T)\, dz_P(t)$$

$$+ \sum_{m=1}^{M} \left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right) dN_{mt}$$

$$- \sum_{m=1}^{M} \lambda_m(t) E_{Nmt}\left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right) dt$$

- K is the number of Brownian factors (for example, 1, 2, 3 or 4).
- The form of the volatility functions $\sigma_{Hk}(t,T)$ can be somewhat general at this time but we assume they are deterministic.
- M is the number of Poisson processes.

- For each k , $k=1,2,\ldots,K$, $dz_{Hk}(t)$ denotes standard Brownian increments. We denote the correlation (assumed constant) between $dz_P(t)$ and $dz_{Hk}(t)$ by ρ_{PHk} , for each k , and the correlation (assumed constant) between $dz_{Hk}(t)$ and $dz_{Hj}(t)$ by ρ_{HkHj} for each k and j .
- $\rho_{HkHj}=1$ if $k=j$

Jump processes

- For each m , $m = 1,\ldots,M$, $\lambda_m(t)$ are the (assumed) deterministic intensity rates of the M Poisson processes.
- $b_m(u)$ for each m are non-negative deterministic functions. We call these the jump decay coefficient functions.
- γ_{mt} are the spot jump amplitudes.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.1 in the paper:
- The spot jump amplitudes are (known) constants. In this case, the jump decay coefficient functions $b_m(u)$ can be non-negative (but otherwise arbitrary) deterministic functions.

Assumptions about the spot jump amplitudes γ_{mt}

- Assumption 2.2 in the paper:
- The spot jump amplitudes are assumed to be independent and identically distributed random variables (assumed independent of everything else). In this case, the jump decay coefficient functions must be equal to zero. ie $b_m(t) \equiv 0$ for all t

Multiple Poisson processes

- All satisfy either Assumption 2.1 or 2.2
- But, if we have more than one Poisson process, we can mix the assumptions
- Eg if 4 Poisson processes, we could have eg 3 satisfying assumption 2.1 and 1 satisfying Assumption 2.2

Implications for completeness

- If all spot jump amplitudes are constants (assumption 2.1) (and there are a sufficient number of futures contracts of different maturities), then the market is complete (and hence the EMM is unique).
- Else the market is not complete (EMM not unique but assume "fixed" by the market).

Implications for arbitrage

- It is not wholly obvious but the assumption of no-arbitrage requires a condition (analogous to the HJM drift condition). This in turn, means we assume random jump amplitudes (assumption 2.2) have an extra condition ie $b_m(t) \equiv 0$
- Bjork, Kabanov and Runggaldier (1997)
- Crosby (2005)

- It is convenient to define:

$$e_m(t,T) \equiv \sum_{m=1}^{M} \lambda_m(t) E_{Nmt}\left(\exp\left(\gamma_{mt} \exp\left(-\int_t^T b_m(u)du \right)\right) - 1 \right)$$

- This is a deterministic quantity.

- Then by Ito: $d\left(\ln H(t,T)\right) =$

$$-\frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^{2}(t,T)+\sigma_{P}^{2}(t,T)-2\sum_{k=1}^{K}\rho_{PHk}\sigma_{P}(t,T)\sigma_{Hk}(t,T)\right\}dt$$

$$-\frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}2\rho_{HkHj}\sigma_{Hk}(t,T)\sigma_{Hj}(t,T)\right\}dt$$

$$+\sum_{k=1}^{K}\sigma_{Hk}(t,T)dz_{Hk}(t)-\sigma_{P}(t,T)dz_{P}(t)$$

$$+\sum_{m=1}^{M}\gamma_{mt}\exp\left(-\int_{t}^{T}b_{m}(u)du\right)dN_{mt}-\sum_{m=1}^{M}e_{m}(t,T)dt$$

Implications

- Gas, Electricity: Short end of futures curve jumps a lot, long end hardly jumps at all (existing models do not seem to have this).
- Gold: Jumps are less of a feature (but they do happen).
- “Gold trades somewhat like a currency”.
- ie jumps cause parallel shift in futures (and forward) curve.

- Also

$$\ln H(t,t) = \ln H(t_0,t) - \int_{t_0}^{t} \frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^2(s,t) + \sigma_P^2(s,t)\right\} ds$$

$$+\int_{t_0}^{t} -\frac{1}{2}\left\{-2\sum_{k=1}^{K}\rho_{PHk}\sigma_P(s,t)\sigma_{Hk}(s,t) + \sum_{k=1}^{K}\sum_{j=1}^{k-1} 2\rho_{HkHj}\sigma_{Hk}(s,t)\sigma_{Hj}(s,t)\right\} ds$$

$$+\int_{t_0}^{t}\sum_{k=1}^{K}\sigma_{Hk}(s,t)\,dz_{Hk}(s) - \int_{t_0}^{t}\sigma_P(s,t)\,dz_P(s)$$

$$+\int_{t_o}^{t}\sum_{m=1}^{M}\gamma_{ms}\exp\left(-\int_{s}^{t} b_m(u)\,du\right) dN_{ms} - \int_{t_0}^{t}\sum_{m=1}^{M} e_m(s,t)\,ds$$

- By differentiating with respect to t , we get the dynamics of $C_t \equiv H(t,t)$,
  but we then find that in general $C_t \equiv H(t,t)$ would be non-Markovian but we would like it to be a Markov process in a finite number of state variables.

- We consider the functional form for the volatilities:

$$\sigma_{Hk}(s,t) = \eta_{Hk}(s) + \chi_{Hk}(s)\exp\left(-\int_s^t a_{Hk}(u)\,du\right)$$

where $\eta_{Hk}(s)$, $\chi_{Hk}(s)$ and $a_{Hk}(u)$

are deterministic functions.

Why?

Gaussian state variables:

- Define the state variables:

$$Y_P(t) = \int_{t_0}^{t} \sigma_r dz_P(s)$$

- And for each k $\quad k = 1,2,\ldots,K$

$$X_{Hk}(t) = \int_{t_0}^{t} \exp\left(-\int_{t_0}^{t} a_{Hk}(u)\,du \right) \chi_{Hk}(s) \exp\left(-\int_{s}^{t_0} a_{Hk}(u)\,du \right) dz_{Hk}(s)$$

$$Y_{Hk}(t) = \int_{t_0}^{t} \eta_{Hk}(s)\,dz_{Hk}(s)$$

Poisson state variables

- Define the state variable (for each m) :

$$X_{Nm}(t) = \int_{t_0}^{t} \gamma_{ms} \exp\left(-\int_{s}^{t} b_m(u)du \right) dN_{ms}$$

- Then with some algebra….

- We have the following expression for the futures commodity price $H(t,T)$ at time t to time T in terms of the initial (ie at time t_0) futures commodity price and the state variables:

$$H\left(t,T\right)=H\left(t_0,T\right)\exp\left(\int_{t_0}^{t}-\frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^{2}\left(s,T\right)+\sigma_{P}^{2}\left(s,T\right)\right\}ds\right)$$

$$\exp\left(\int_{t_0}^{t}-\frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}2\rho_{HkHj}\sigma_{Hk}\left(s,T\right)\sigma_{Hj}\left(s,T\right)-\sum_{k=1}^{K}2\rho_{PHk}\sigma_{P}\left(s,T\right)\sigma_{Hk}\left(s,T\right)\right\}ds\right)$$

$$\exp\left(\frac{\exp\left(-\alpha_r\left(T-t\right)\right)}{\alpha_r}X_P\left(t\right)-\frac{1}{\alpha_r}Y_P\left(t\right)\right)$$

$$\exp\left(\sum_{k=1}^{K}Y_{Hk}\left(t\right)+\sum_{k=1}^{K}\left[\exp\left(-\int_{t}^{T}a_{Hk}\left(u\right)du\right)X_{Hk}\left(t\right)\right]\right)$$

$$\exp\left(\sum_{m=1}^{M}\left(\left\{\exp\left(-\int_{t}^{T}b_m\left(u\right)du\right)X_{Nm}\left(t\right)\right\}-\int_{t_0}^{t}e_m\left(s,T\right)ds\right)\right)$$

Convenience Yields

- We show in the paper that:
- Our model automatically generates stochastic convenience yields.
- The convenience yields also exhibit jumps, except in the special case that $b_m(t) \equiv 0$ for all m
- We do not need to make any assumptions about the stochastic process for convenience yields or its market price of risk.

Mean reversion in spot commodity prices

- In the paper, we show that the value of the commodity ie $C_t \equiv H(t,t)$ follows a mean-reverting jump-diffusion process.
- We also show how the jump decay coefficient functions $b_m(t)$, if they are non-zero (ie assumption 2.1 only), can also contribute to the mean reversion effect.
- Jump decay coefficient functions analogous to mean reversion rates.

Monte Carlo Simulation

- How can we simulate futures commodity prices in this model?
- About 3 slides ago I wrote down an expression for the futures commodity price in terms of its initial value and the state variables:

$$H\left(t,T\right)=H\left(t_0,T\right)\exp\left(\int_{t_0}^{t}-\frac{1}{2}\left\{\sum_{k=1}^{K}\sigma_{Hk}^{2}\left(s,T\right)+\sigma_{P}^{2}\left(s,T\right)\right\}ds\right)$$

$$\exp\left(\int_{t_0}^{t}-\frac{1}{2}\left\{\sum_{k=1}^{K}\sum_{j=1}^{k-1}2\rho_{HkHj}\sigma_{Hk}\left(s,T\right)\sigma_{Hj}\left(s,T\right)-\sum_{k=1}^{K}2\rho_{PHk}\sigma_{P}\left(s,T\right)\sigma_{Hk}\left(s,T\right)\right\}ds\right)$$

$$\exp\left(\frac{\exp\left(-\alpha_r\left(T-t\right)\right)}{\alpha_r}X_P\left(t\right)-\frac{1}{\alpha_r}Y_P\left(t\right)\right)$$

$$\exp\left(\sum_{k=1}^{K}Y_{Hk}\left(t\right)+\sum_{k=1}^{K}\left[\exp\left(-\int_{t}^{T}a_{Hk}\left(u\right)du\right)X_{Hk}\left(t\right)\right]\right)$$

$$\exp\left(\sum_{m=1}^{M}\left(\left\{\exp\left(-\int_{t}^{T}b_m\left(u\right)du\right)X_{Nm}\left(t\right)\right\}-\int_{t_0}^{t}e_m\left(s,T\right)ds\right)\right)$$

Monte Carlo Simulation

- Therefore, we can easily do Monte Carlo simulation if we can simulate the state variables:
- Gaussian state variables are straightforward.
- So lets focus on the Poisson state variables.

Poisson

- By definition, the probability $Q_m(t_0, t; n_m)$ that there are n_m jumps on the Poisson Process N_{mt} in the time period t_0 to t is:

$$Q_m(t_0, t; n_m) = \exp\left(-\int_{t_0}^{t} \lambda_m(u)\,du\right) \frac{\left[\int_{t_0}^{t} \lambda_m(u)\,du\right]^{n_m}}{n_m!}$$

- We also need the following result (an extension of a result in Karlin and Taylor (1975) “A first course in stochastic processes”):

Proposition

- Suppose that we know that there have been n_m jumps between time t_0 and time t. Write the arrival times of the jumps as $S_{1m}, S_{2m}, \ldots, S_{n_m m}$. The conditional joint density function of the arrival times, when the arrival times are viewed as unordered random variables, conditional on $N_{mt} = n_m$ is:

$$\Pr\left(S_{1m} = s_{1m} \,\&\, S_{2m} = s_{2m} \,\&\ldots\&\, S_{n_m m} = s_{n_m m} \mid N_{mt} = n_m\right) =$$

$$\frac{\left[\lambda_m\left(s_{1m}\right)\right]\left[\lambda_m\left(s_{2m}\right)\right]\ldots\left[\lambda_m\left(s_{n_m m}\right)\right]}{\left(\left[\int_{t_0}^{t} \lambda_m(u)\,du\right]^{n_m}\right)}$$

- (As an aside, if intensity rates are all constants => uniform)
- Simulate n_m by simulating Poisson.
- Simulate arrival times.

- Then Poisson state variable:

$$X_{Nm}(t) = \int_{t_0}^{t} \gamma_{ms} \exp\left(-\int_{s}^{t} b_m(u)du \right) dN_{ms}$$

$$\equiv \sum_{i=1}^{n_m} \gamma_{mS_{im}} \exp\left(-\int_{S_{im}}^{t} b_m(u)du \right)$$

Poisson state variables

- Hence we can simulate the Poisson state variables.
- Hence we can simulate futures commodity prices.
- Also there is no discretisation error bias in the Monte Carlo simulation.

Assumption 2.2

- The only assumption, in the case of assumption 2.2, that we have made thus far is that the spot jump amplitudes are independent and identically distributed.

Assumption 2.2

- We now specialise assumption 2.2 and assume that the spot jump amplitudes, for each m, are normally distributed with mean β_m and standard deviation υ_m.

Assumption 2.1

- Assumption 2.1 is unchanged. We write the known constant spot jump amplitude as β_m

Pricing of standard options

- We would like to price standard (plain vanilla) European options in a computationally efficient manner
- (Might allow us to get implied parameters from market prices of options).
- However, futures commodity prices are NOT log-normally distributed.

- The key to pricing standard European options is the following observation:
- CONDITIONAL on the number of jumps and their arrival times, futures commodity prices ARE log-normally distributed.
- Then bring standard results into play (Merton (1976) and Jarrow and Madan (1995))

- We wish to price, at time t, a European (non-path-dependent) option maturing at time T_1, written on the futures commodity price, where the futures contract matures at time T_2 (with $t \leq T_1 \leq T_2$)
- The payoff of the option at time T_1 is

$$D(H(T_1, T_2))$$

- Conditional on the number of jumps n_m, for $m = 1,\dots,M$ and the arrival times $s_{1m}, s_{2m},\dots, s_{n_m m}$ of these jumps, the value of the option at time t is:

$$Exp_t\left[\exp\left(-\int_t^{T_1} r(u)du\right)D(H(T_1,T_2)) \mid n_m, s_{1m}, s_{2m},\dots s_{n_m m}\right]$$

- So using standard results about conditional expectations and the results earlier:

- The price of the option at time t is:

$$\sum_{n_1=0}^{n_1=\infty}\sum_{n_2=0}^{n_2=\infty}\sum_{n_M=0}^{n_M=\infty} Q_1(t,T_1;n_1)Q_2(t,T_1;n_2)\ldots Q_M(t,T_1;n_M)$$

$$\int_t^{T_1}\int_t^{T_1}\ldots\int_t^{T_1} Exp_t\left[\exp\left(-\int_t^{T_1} r(u)du\right)D(H(T_1,T_2)) \mid n_m, s_{1m}, s_{2m},\ldots s_{n_m m}\right]$$

$$\prod_{m=1}^{M}\frac{[\lambda_m(s_{1m})][\lambda_m(s_{2m})]\ldots[\lambda_m(s_{n_m m})]}{\left(\left[\int_{t_0}^{t}\lambda_m(u)du\right]^{n_m}\right)} ds_{11}ds_{21}\ldots ds_{n_1 1}\ldots ds_{n_M M}$$

Standard European Options on futures

- To obtain the form for the price of a standard European options on futures, whose payoff is $\max\left(\eta\left(H\left(T_1,T_2\right)-K,0\right)\right)$

Replace

$$Exp_t\left[\exp\left(-\int_t^{T_1} r(u)du\right)D\left(H\left(T_1,T_2\right)\right)\mid n_m, s_{1m}, s_{2m}, \ldots s_{n_m m}\right]$$

by:

Standard European options on futures

$$\eta P(t,T_1)$$

$$\left(H(t,T_2)V(t,T_1;n_m;T_2,M)\exp\left(\int_t^{T_1} A(s,T_1,T_2)ds\right)N(\eta d_1) - KN(\eta d_2)\right)$$

- (looks like Black (1976) formula)

- Where

$$A(s,T_1,T_2)\equiv\left(\sum_{k=1}^{K}\rho_{PHk}\sigma_P(s,T_1)\sigma_{Hk}(s,T_2)\right)-\sigma_P(s,T_1)\sigma_P(s,T_2)$$

- (note this term is simply because of the stochastic interest-rates – it is the instantaneous covariance between bond prices and futures commodity prices)

- And (this term depends on number of jumps and their arrival times)

$$V\left(t, T_1; n_m; T_2, M\right) \equiv$$

$$\exp\left(\sum_{m=1}^{M}\left\{1_{m(2.1)}\sum_{i=1}^{n_m}\beta_m \exp\left(-\int_{s_{im}}^{T_2} b_m(u)du\right) + 1_{m(2.2)} n_m\left(\beta_m + \frac{1}{2}\upsilon_m^2\right)\right\}\right)$$

$$\exp\left(-\sum_{m=1}^{M}\left(\int_{t}^{T_1} e_m(s, T_2)ds\right)\right)$$

Options continued

- Also we can semi-analytically value:
- Futures-style options (both European and American) on futures contracts.
- Note that many exchange traded options are of this type and exchange traded options are often the most liquid and have the smallest bid-offer spreads.

Options continued

- We can also semi-analytically value:
- Standard European options on forwards.
- Standard European options on the spot.

Options continued

$$\sum_{n_1=0}^{n_1=\infty}\sum_{n_2=0}^{n_2=\infty}\sum_{n_M=0}^{n_M=\infty} Q_1\left(t,T_1;n_1\right)Q_2\left(t,T_1;n_2\right)\ldots Q_M\left(t,T_1;n_M\right)$$

$$\int_t^{T_1}\int_t^{T_1}\ldots\int_t^{T_1} Exp_t\left[\exp\left(-\int_t^{T_1} r(u)du\right)D\left(H\left(T_1,T_2\right)\right) \mid n_m, s_{1m}, s_{2m}, \ldots s_{n_m m}\right]$$

$$\prod_{m=1}^{M}\frac{\left[\lambda_m\left(s_{1m}\right)\right]\left[\lambda_m\left(s_{2m}\right)\right]\ldots\left[\lambda_m\left(s_{n_m m}\right)\right]}{\left(\left[\int_{t_0}^{t}\lambda_m(u)du\right]^{n_m}\right)} ds_{11}ds_{21}\ldots ds_{n_1 1}\ldots ds_{n_M M}$$

- Lets look at this formula in more depth.
- Poisson mass functions rapidly tend to zero once the number of jumps >= mean number of jumps (=> can truncate infinite sum).
- Need to integrate over arrival times. How?

- Lets look at this formula in more depth.
- Poisson mass functions rapidly tend to zero once the number of jumps >= mean number of jumps
- Need to integrate over arrival times. How?
- Monte Carlo - We call this the MCIATJ methodology in the paper.

- Use Monte Carlo to simulate the arrival times of the jumps, conditional on the number of jumps.
- In the special case that the intensity rates are constants (particularly straightforward), then the arrival times are uniform on $[t, T_1]$
- (if NOT use inverse transformation method).

- It sounds computationally intensive but it isn’t.
- In the paper, we price 30 options of 5 different strikes and 6 different maturities
- We use anti-thetic variates.
- We use a Control Variate (see paper for details).

- The options are all standard European calls on futures prices.
- We demonstrate in the paper that it is possible to price all 30 options in < 0.51 seconds
- This is < 0.017 seconds per option
- All standard errors < 0.003 % of spot (mostly even less than 0.001 % of spot).
- MCIATJ is fast for short-dated options.

Calibration

- This means we can obtain the model parameters from the market prices of options by doing a least-square fit.
- MCIATJ relatively slow for long-dated options.
- Is there an even faster way of pricing standard European options?

Fourier Transforms

- Many authors have shown that it is possible to price standard European options easily if the characteristic function of the terminal asset price is known analytically.
- Carr and Madan (1999), Heston (1993), Lee (2004), Sepp (2003), Duffie et al. (2000)
- Characteristic function is Fourier Transform of the probability density function.

Problem

- In our model, the characteristic function is not analytic if any of the Poisson processes satisfy Assumption 2.1 with $b_m(t) > 0$ (can be expressed as an integral but it involves sines and cosines => tricky double (oscillatory) integral).

FTPS methodology

- However, we show that it is possible to derive a rapidly convergent power series expansion for terms in the characteristic function which converges to (say) 10^{-11} accuracy (or better) after typically about 10 to 40 terms in the series.
- Therefore, very fast, accurate and easy on a computer.
- We call this the FTPS methodology.

- Multiply FT of payoff by characteristic function. A single one-dimensional integral (using eg Simpson’s rule) then gives the option price.
- This is true even with multiple Brownian motions and multiple Poisson processes.

With FTPS (Fourier Transform) methodology:

- Can price the same 30 options in < 0.016 seconds (MCIATJ was 0.51 seconds)
- => less than 1 millisecond per option.
- Can trade time against accuracy.
- But our FTPS (Fourier Transform) methodology is at least 10 times (usually even better) more accurate than the Monte Carlo integration over the arrival times of the jumps (MCIATJ methodology).

Summary

- The model is arbitrage-free.
- Automatically fits initial futures (or forward) commodity price curve.
- Spot price exhibits mean reversion.
- Long-dated futures can jump less than short-dated futures.
- Generates stochastic convenience yields without further ado.

Summary cont’d

- Can price exotics via Monte Carlo.
- Can price standard European options via:
- A Monte Carlo based (MCIATJ) algorithm or Fourier Transform based (FTPS) algorithm.
- FTPS is faster and more accurate.

Suggestions for further research 1

- Asian commodity options (also called commodity swaptions) – approximate Characteristic Function ??.
- Examine whether or not arbitrage is possible in case of exponentially dampened jumps AND random jump amplitudes.

Suggestions for further research 2

- American Options? PDE based approach? Fourier Transform approach?
- Spread options? See Dempster and Hong (2000) Judge Institute of Management Working paper WP26/2000 (published in: Proceedings of the First World Congress of the Bachelier Finance Society, Paris (2000)).